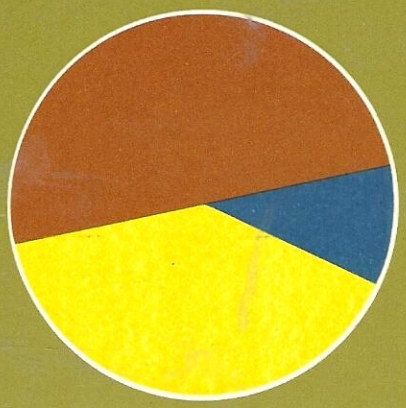
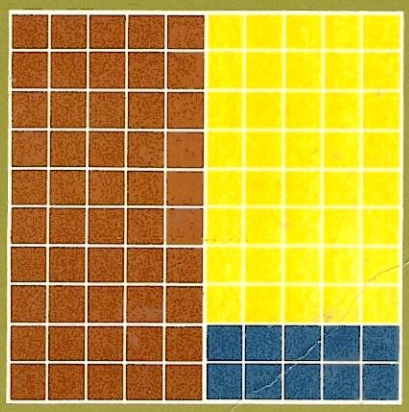
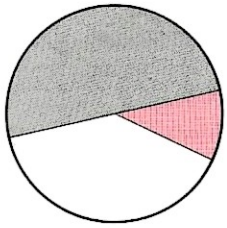
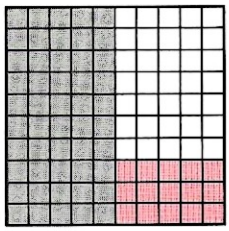
ELEMENTARY SCHOOL MATHEMATICS

BOOK 6









BOOK 6

ELEMENTARY SCHOOL MATHEMATICS

ROBERT E. EICHOLZ | *Ball State University*

PHARES G. O'DAFFER | *Ball State University*

Consulting authors

CHARLES F. BRUMFIEL | *The University of Michigan*

MERRILL E. SHANKS | *Purdue University*

ADDISON-WESLEY · PALO ALTO, CALIFORNIA

Book 6

Table of Contents

1 PLACE VALUE AND NUMBER BASES 1

General concept of place value
Reading and writing of large numbers
Inequalities
Exponents and powers of ten
Rounding of large numbers
Expanded notation using exponents
Other systems of numeration
Measurement lesson 1/Length and perimeter

2 WHOLE NUMBERS 22

Basic number facts
Relationships between operations
General concepts for the operations
Basic principles
Special products and quotients
Work with function concepts
Measurement lesson 2/Area and surface area

3 COMPUTING 44

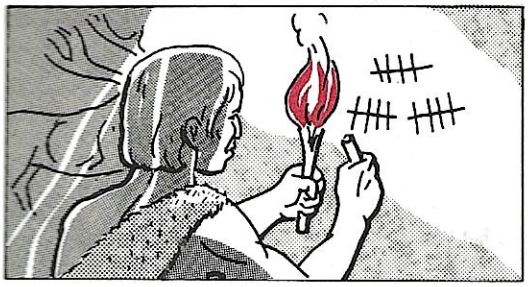
Review of the addition, subtraction, and multiplication algorithms
Estimation
Extension of the division algorithm
Short division
Word problems involving use of the algorithms
Averages
Measurement lesson 3/Volume

4

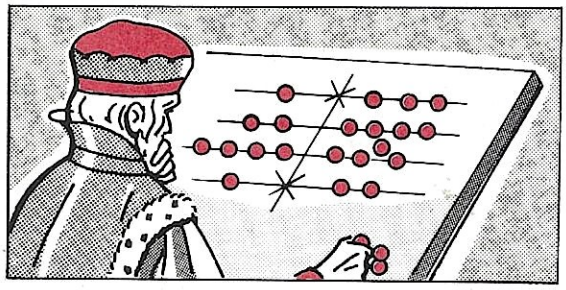
ADDISON-WESLEY PUBLISHING COMPANY, INC.
PALO ALTO · Reading, Massachusetts · London
New York · Dallas · Atlanta · Barrington, Illinois

. 308
. 326
. 329
. 330

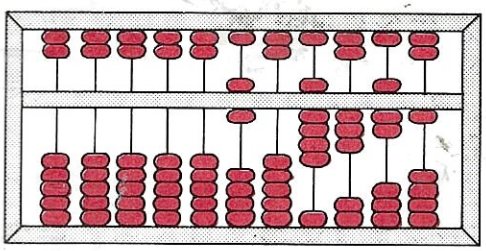
CHAPTER 1 | Place value and number bases



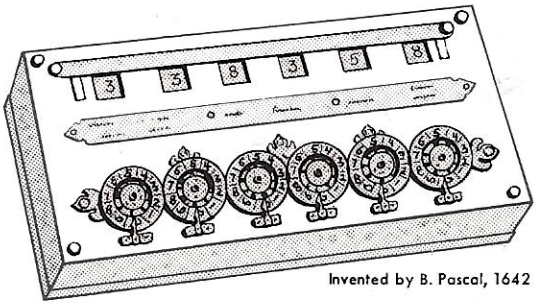
Explain why the cave man may have used groups of 5 to keep his hunting record.



Explain how this early merchant shows 1241 and 3482 on the counting board. A red marker between the lines replaces 5 marks on the line below it.



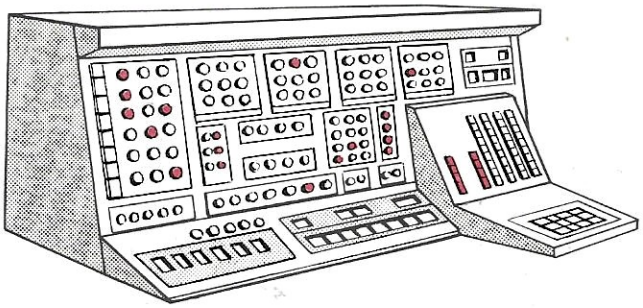
The Chinese abacus (called a **suan-pan**) shows the number 609,371. Explain this.



Invented by B. Pascal, 1642

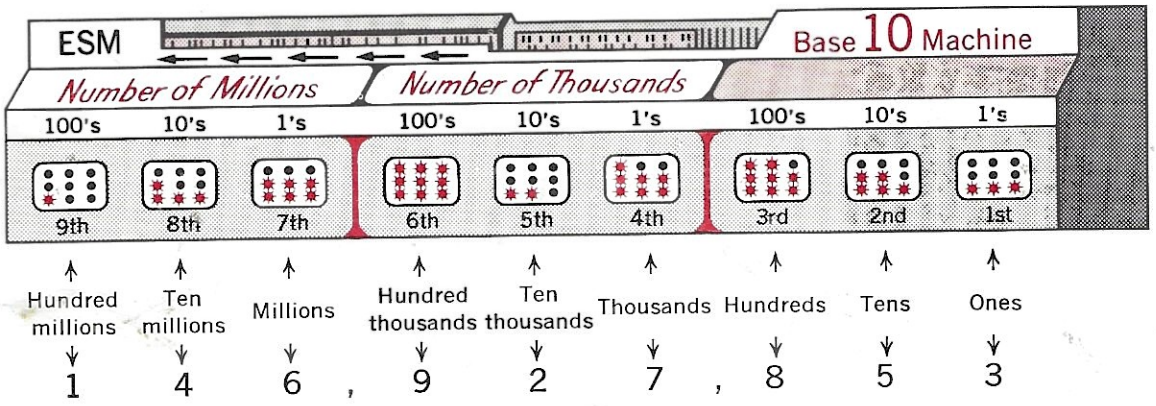
Explain how this first calculating machine uses place value. At each complete turn of any wheel, the wheel to its left turns $\frac{1}{10}$ of the way around.

Modern computers like this, using base two, can do many thousands of multiplications in one second. In the base ten system, only 10 digits are used to write numerals. How many digits are needed in base two?



Throughout the ages man has been able to make great progress in computing because of the concept of **place value**. This chapter will help you to better understand this important idea.

Place value in base ten



The lights on the machine show the number a card has signaled the machine to remember. The numeral is displayed below the machine to point out the use of **place value** in writing symbols for whole numbers. Notice that commas are used to separate the digits into groups of three, beginning at the right. Each group of three digits is called a **period**.

EXERCISES

1. Counting from right to left, we call the first place the **ones** place, the second place the **tens** place, and so on. Study the picture above and give the name of each of these places:

   [A] 3rd [B] 4th [C] 5th [D] 6th [E] 7th [F] 8th [G] 9th

2. Give the names of the 10th, 11th, and 12th places.

3. In the numeral 534,896,201 tell what digit is in each of these places:

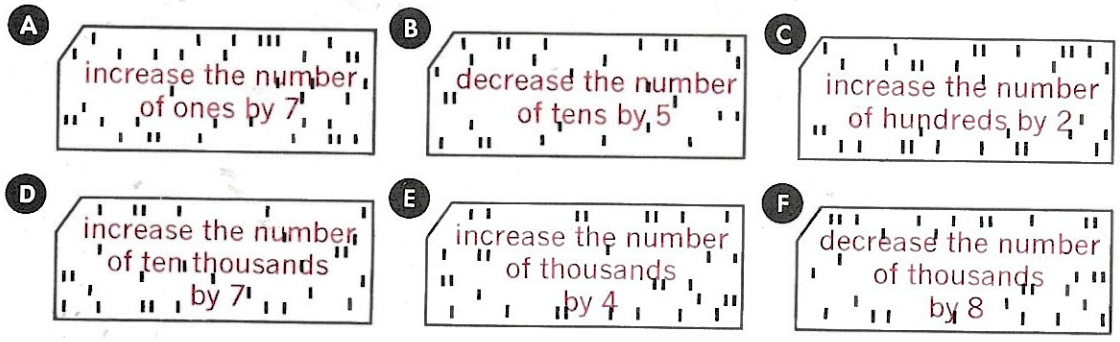
   [A] ones [B] thousands [C] millions [D] tens [E] ten thousands
   [F] ten millions [G] hundreds [H] hundred thousands [I] hundred millions

4. Starting at the right, the **first period** of the numeral under the machine contains the digits 8, 5, and 3. Copy and complete these sentences about the numeral under the machine.

   [A] The **second period** contains the digits _?_ and tells how many _?_.

   [B] The **third period** contains the digits _?_ and tells how many _?_.

5. When any one of the cards below is put into the base ten machine on page 2, it gives the machine the signal shown in red.


A
increase the number of ones by 7
B
decrease the number of tens by 5
C
increase the number of hundreds by 2
D
increase the number of ten thousands by 7
E
increase the number of thousands by 4
F
decrease the number of thousands by 8

[A] If card A were put into the machine, how many lights would be on in the 1st place? the 2nd place?

[B] If card B were put into the machine, how many lights would be on in the 2nd place? the 3rd place?

[C] If card C were put into the machine, how many lights would be on in the 3rd place? the 4th place?

[D] If card D were put into the machine, how many lights would be on in the 5th place? the 6th place?

[E] If card E were put into the machine, how many lights would be on in the 4th place? the 5th place? the 7th place?

[F] If card F were put into the machine, how many lights would be on in the 5th place? the 4th place?

6. Copy the sentence. Give the missing words and numbers.

[A] 687: The 8 in the _?_ place means $8 \times$ ▒.

[B] 687: The 6 in the _?_ place means $6 \times$ ▒.

[C] 395,687: The 5 in the _?_ place means $5 \times$ ▒.

[D] 395,687: The 9 in the _?_ place means $9 \times$ ▒.

[E] 395,687: The 3 in the _?_ place means $3 \times$ ▒.

[F] 214,395,687: The 4 in the _?_ place means $4 \times$ ▒.

[G] 214,395,687: The 1 in the _?_ place means $1 \times$ ▒.

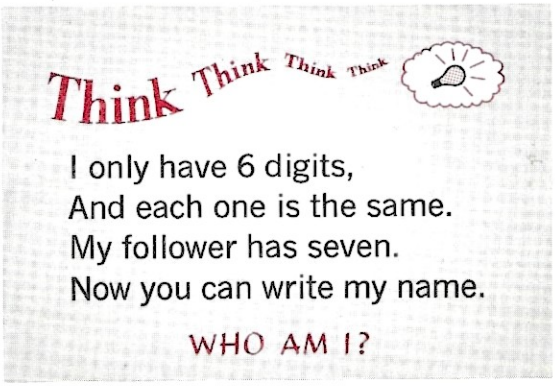
[H] 214,395,687: The 2 in the _?_ place means $2 \times$ ▒.

[I] 214,395,687: The 7 in the _?_ place means $7 \times$ ▒.

Place-value exercises

1. Solve the equations.

[A] $436 = (4 \times n) + (3 \times 10) + 6$
[B] $584 = (5 \times 100) + (8 \times n) + 4$
[C] $689 = (6 \times 100) + (8 \times 10) + n$
[D] $n = (7 \times 100) + (3 \times 10) + 2$
[E] $4056 = (4 \times n) + (5 \times 10) + 6$
[F] $7308 = (7 \times n) + (3 \times 100) + 8$
[G] $8053 = (8 \times 1000) + (5 \times n) + 3$
[H] $54{,}317 = (5 \times n) + (4 \times 1000) + (3 \times 100) + (1 \times 10) + 7$
[I] $895{,}603 = (8 \times n) + (9 \times 10{,}000) + (5 \times 1000) + (6 \times 100) + 3$
[J] $n = (3 \times 10{,}000{,}000) + (7 \times 1{,}000{,}000) + (5 \times 100{,}000) + (6 \times 100)$



2. In the example below, the number is written using **expanded notation**. For each exercise, use expanded notation to represent the number.

Example: $93{,}564 = (9 \times 10{,}000) + (3 \times 1000) + (5 \times 100) + (6 \times 10) + 4$

[A] 5634
[B] 9847
[C] 54,627
[D] 67,491
[E] 270,423
[F] 869,070
[G] 6,893,451
[H] 4,563,782

3. Solve the equations.

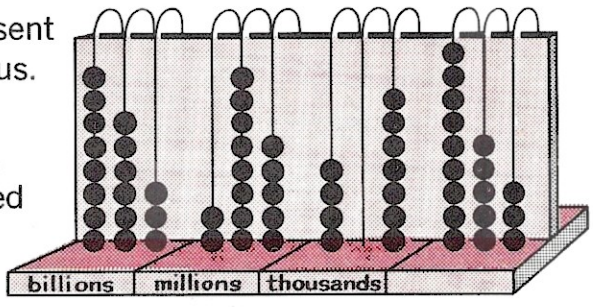
[A] $10 \times 10 = n$
[B] $100 \times 10 = n$
[C] $(10 \times 10) \times 10 = n$
[D] $1000 \times 10 = n$
[E] $(10 \times 10 \times 10) \times 10 = n$
[F] $10 \times 10 \times 10 \times 10 \times 10 = n$

4. Solve the equations.

[A] $5648 = (5 \times 10 \times 10 \times 10) + (n \times 10 \times 10) + (4 \times 10) + 8$
[B] $67{,}400 = (6 \times 10 \times 10 \times 10 \times n) + (7 \times 10 \times 10 \times 10) + (4 \times 10 \times 10)$

★**5.** [A] Using expanded notation, represent the number shown on this abacus.

[B] Write the number that results when the numbers of hundreds, hundred thousands, and hundred millions shown on the abacus are each increased by 8.



Inequality exercises

1. Give the correct sign (<, >, or =) for each ◯.

[A] $7659 \bigcirc (7 \times 1000) + (6 \times 100) + (4 \times 10) + 9$

[B] $63{,}742 \bigcirc (6 \times 10{,}000) + (3 \times 1000) + 742$

[C] $59{,}406 \bigcirc (6 \times 10{,}000) + (0 \times 1000) + 406$

[D] $378{,}941 \bigcirc (3 \times 100{,}000) + (78 \times 1000) + 940$

[E] $8{,}406{,}000 \bigcirc (8 \times 1{,}000{,}000) + (4 \times 100{,}000) + (6 \times 10{,}000)$

2. Solve the equations.

[A] $847{,}618 = 840{,}618 + n$

[B] $642{,}935 = 542{,}935 + n$

[C] $801{,}263 = 800{,}263 + n$

[D] $543{,}006 = 543{,}106 - n$

[E] $727{,}342 = 700{,}342 + n$

[F] $586{,}419{,}379 = 576{,}419{,}379 + n$

[G] $864{,}907{,}426 = 864{,}906{,}426 + n$

[H] $570{,}436{,}329 = 572{,}436{,}329 - n$

[I] $687{,}498{,}593 = 687{,}400{,}593 + n$

[J] $642{,}000{,}723 = 642{,}020{,}723 - n$

3. Give the correct sign (<, >, =) for each ◯. Do not find the sums.

[A] $35 + 46 \bigcirc 34 + 46$

[B] $529 + 467 \bigcirc 529 + 477$

[C] $853 + 297 \bigcirc 863 + 287$

[D] $3748 + 2983 \bigcirc 3848 + 2983$

[E] $5764 + 3488 \bigcirc 5764 + 3498$

[F] $83{,}426 + 549 \bigcirc 83{,}326 + 649$

4. [A] How many thousands make a million?

[B] How many hundred thousands make a million?

5. These exercises are about the number 342,817,695.

[A] Rearrange the digits to name the largest number possible.

[B] Rearrange the digits to name the smallest number possible.

[C] Give the number 5 million greater.

[D] Give the number 42 million less.

★[E] Give the number a half million less.

★[F] What must be added to the number to reach 343 million?

Think Think Think Think

How much money would you lose if you traded a thousand million dollars for a million thousand dollars?

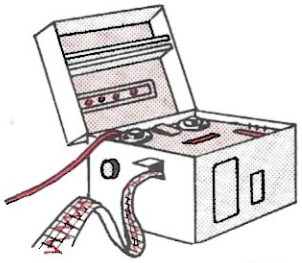
Write the numeral for a million thousand.

How large is a billion?

CALENDAR 1902 MAY						
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

1. One million minutes is almost 2 years.
One billion minutes is about 1902 years.

2. If your heart beats 70 times each minute, it would take about 10 days for it to beat one million times and about 27 years for it to beat one billion times.



3. 400 classes, each containing 30 sixth-graders, would weigh about one million pounds. It would take 400,000 such classes to weigh one billion pounds.

4. One million inches is almost 16 miles.
One billion inches is about 16,000 miles.
You would travel about 16,000 miles if you made 5 trips from New York to San Francisco, or if you went two thirds of the way around the world.

EXERCISES

One billion is 1000 million. We write: **1,000**,000,000

Write the number that is:

1. 1 billion more than 1 billion
2. 2 billion more than 1 billion
3. 1 billion more than 9 billion
4. 5 billion more than 27 billion
5. 100 million more than 1 billion
6. 10 million more than 1 billion
7. 1 million more than 1 billion
8. 1 thousand more than 1 billion
9. 1 more than 1 billion
10. 1 less than 1 billion

★11. One half as much as 1 billion

★12. 101 less than 1 billion

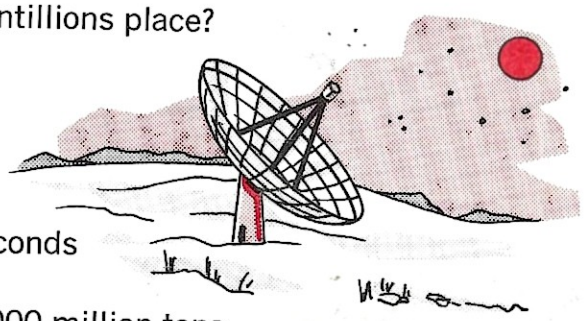
Larger numbers

The diagram below shows the names of some of the periods. It should help you read the large number given.

Sextillions	*Quintillions*	*Quadrillions*	*Trillions*	*Billions*	*Millions*	*Thousands*	
↓	↓	↓	↓	↓	↓	↓	
476,	587,	319,	608,	742,	871,	023,	964

DISCUSSION EXERCISES

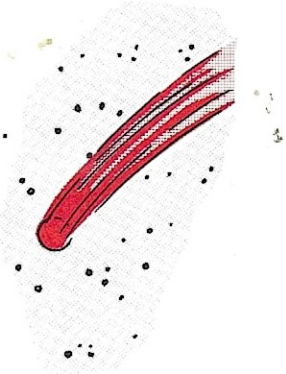
1. [A] Read the number given above.
   [B] What digit is in the hundred quintillions place?



2. Read each of the following facts.
   [A] Diameter of the sun:
   4,567,000,000 feet
   [B] Estimated life of the sun:
   100,000,000,000,000,000 seconds
   [C] Weight of the sun:
   2,000,000,000,000,000,000,000 million tons
   [D] Average distance from earth to sun:
   92,955,750 miles
   [E] Recent estimate of the age of the earth:
   1,806,750,000,000 days
   [F] Weight of the earth:
   5,887,613,230,000,000,000 thousand tons

EXERCISES

1. The distance from the earth to the sun is a number in the millions. Which number given in the discussion exercises is in the:
   [A] billions? [B] trillions? [C] quadrillions?
   [D] quintillions? [E] sextillions?



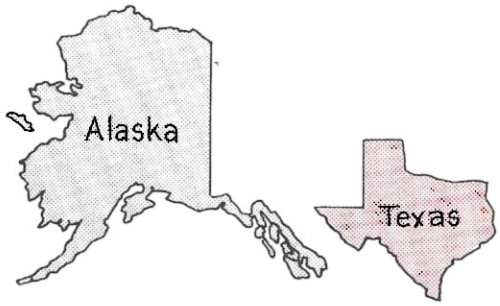
2. The nearest star beyond our sun is called **Proxima Centauri**. It is about 25 trillion miles away. Write this numeral.
3. Sirius is the brightest star in the heavens. It is 50 trillion miles away, and its weight has been calculated to be 45,800 sextillion tons. Write each of these numerals.
4. Which weighs more, the earth or the sun?

Approximation–rounding

In a recent census the population of the United States was given as 179,323,175. When less accurate information is needed, we often use a number that is "close" or approximately equal to the exact population. We say that the population is:

179,323,175
or 179,323,180 (rounded to the nearest ten)
or 179,323,200 (rounded to the nearest hundred)
or 179,323,000 (rounded to the nearest thousand)
or 179,320,000 (rounded to the nearest ten thousand)
or 179,300,000 (rounded to the nearest hundred thousand)
or 179,000,000 (rounded to the nearest million)
or 180,000,000 (rounded to the nearest ten million)
or 200,000,000 (rounded to the nearest hundred million)

EXERCISES



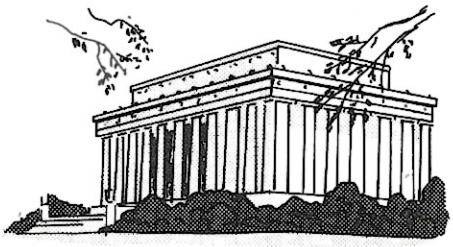
1. Alaska and Texas are the two largest states in the United States. The area of Alaska is 571,065 square miles. The area of Texas is 262,840 square miles. Give each area rounded to the nearest thousand.

2. One of the first space capsules reached a top speed of 17,546 miles per hour. Give this speed:

[A] rounded to the nearest ten

[B] rounded to the nearest hundred

[C] rounded to the nearest thousand



3. The Lincoln Memorial in Washington, D.C., cost a total of $3,045,400. Give the cost:

[A] to the nearest ten thousand [B] to the nearest million

4. There were 79,830,600 telephones in North America in a recent year. Give this number:

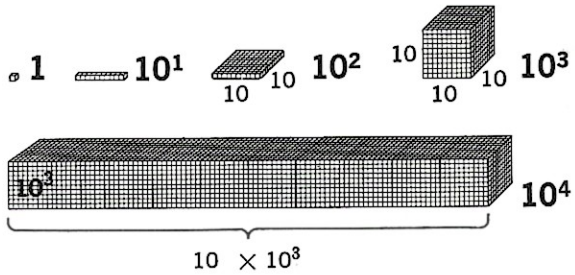
[A] to the nearest thousand [B] to the nearest million

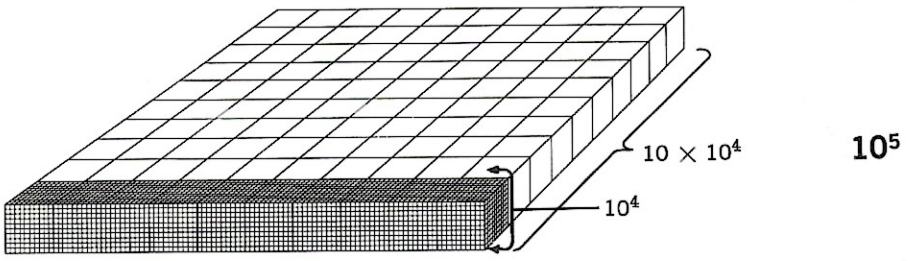
Exponents

The table below shows how we use exponents to simplify the writing of expressions with several factors.

For this product	We think	We write	We say
10×10	10 used as a factor 2 times	10^2	10 to the 2nd power, or 10 squared
$10 \times 10 \times 10$	10 used as a factor 3 times	10^3	10 to the 3rd power, or 10 cubed
$10 \times 10 \times 10 \times 10$	10 used as a factor 4 times	10^4	10 to the 4th power
$10 \times 10 \times 10 \times 10 \times 10$	10 used as a factor 5 times	10^5	10 to the 5th power
$10 \times 10 \times 10 \times 10 \times 10 \times 10$	10 used as a factor 6 times	10^6	10 to the 6th power

The numbers 10^2, 10^3, 10^4, and so on are called **powers of ten.** Ten is called the **base**. The numbers indicated in red are called **exponents.** 10^1 means 10.





Study the figures above and tell how you think we might picture 10^6.

EXERCISES

1. Give the missing numbers.

[A] $10^2 =$ ▒▒ [C] $10^4 =$ ▒▒ [E] $10^6 =$ ▒▒ [G] $10^8 =$ ▒▒
[B] $10^3 =$ ▒▒ [D] $10^5 =$ ▒▒ [F] $10^7 =$ ▒▒ [H] $10^9 =$ ▒▒

2. Find the products.

[A] 10×10 [C] $10^1 \times 10^2$ [E] $10^3 \times 10$ [G] $10^4 \times 10$ [I] $10^4 \times 10^3$
[B] $10^2 \times 10^1$ [D] $10^2 \times 10^2$ [F] $10^1 \times 10^3$ [H] $10^4 \times 10^2$ [J] $10^3 \times 10^2$

3. Give the number for **a**. Then give the product for **b**.

[A] $5 \times 10^3 = 5 \times \mathbf{a} = \mathbf{b}$
[B] $12 \times 10^3 = 12 \times \mathbf{a} = \mathbf{b}$
[C] $37 \times 10^2 = 37 \times \mathbf{a} = \mathbf{b}$
[D] $3 \times 10^4 = 3 \times \mathbf{a} = \mathbf{b}$

4. Give the number for **a**. Then give the power of ten for **b**.

[A] $300 = 3 \times \mathbf{a} = 3 \times \mathbf{b}$
[B] $6000 = 6 \times \mathbf{a} = 6 \times \mathbf{b}$
[C] $2700 = 27 \times \mathbf{a} = 27 \times \mathbf{b}$
[D] $90{,}000 = 9 \times \mathbf{a} = 9 \times \mathbf{b}$

5. Find the products.

[A] 4×10^3 [C] 13×10^3 [E] 4×10^4 [G] 57×10^2 [I] 8×10^5
[B] 8×10^2 [D] 48×10^2 [F] 65×10^3 [H] 39×10^3 [J] 7×10^6

6. Write each of the following as a number less than ten times a power of ten.

[A] 700 (Answer: 7×10^2) [B] 900 [C] 8000 [D] 200 [E] 6000 [F] 70,000 [G] 90,000 [H] 20,000 [I] 600,000

Think Think Think Think

Suppose a book has 100 names listed in each column, with 5 columns per page. The book has 200 pages. How many books are needed to list all the people in the United States, if the population is 200 million?

$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$
⋮

The number 10^{100} is sometimes called a **googol**. How many digits would you need to write the ordinary base ten numeral for one googol?

More about exponents

The red numerals will help you see an easy way to give the ordinary base ten numeral for any power of ten. They will also help you write an ordinary numeral as a power of ten. Check each product.

$10^1 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1000$

$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$

$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000$

EXERCISES

1. Write the ordinary base ten numeral for each exercise.

[A] 10^7 [C] 10^9 [E] 5×10^3 [G] 7×10^6 [I] 24×10^2

[B] 10^8 [D] 10^{10} [F] 8×10^2 [H] 6×10^4 [J] 38×10^3

2. Write each of the following as a number less than ten times a power of ten.

[A] 5000 [C] 70,000 [E] 300,000 [G] 7 million ★[I] 6 sextillion

[B] 7000 [D] 20,000 [F] 5,000,000 [H] 80 million ★[J] 500 quadrillion

3. Write each of the following, using the expanded notation (exponent form) illustrated in the example.

[Example: $362 = (3 \times 10^2) + (6 \times 10) + (2 \times 1)$]

[A] 457 [B] 348 [C] 4265 [D] 37,582 ★[E] 7,638,423

★**4.** These numbers have been written in expanded notation and the addends have been rearranged. Copy each exercise and give the missing powers of ten.

[A] $3657 = (6 \times \square) + (5 \times \square) + (3 \times \square) + (7 \times 1)$

[B] $57{,}040 = (4 \times \square) + (7 \times \square) + (5 \times \square)$

[C] $345{,}620 = (5 \times \square) + (2 \times \square) + (6 \times \square) + (4 \times \square) + (3 \times \square)$

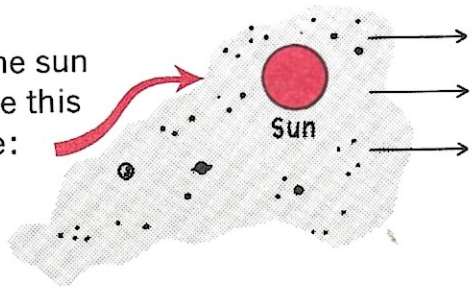
[D] $654{,}000{,}000 = (5 \times \square) + (6 \times \square) + (4 \times \square)$

The universe

1. The diameter of our sun is about 865,000 miles. It would take over a million earths to make an object the size of the sun.
2. The earth is about 93,000,000 miles from the sun.
3. One of the two nearest stars, Alpha Centauri, is like our sun and is about 24,745,000,000,000 miles away.
4. There are probably 100,000,000,000 other stars in our galaxy, or group of stars.
5. As many as 500,000,000,000 other galaxies (groups of stars) have been detected by high-powered telescopes.

The facts above and the diagram below will give you some idea of the size of the universe.

If the sun were this size:



The earth would be a speck about 3 feet away.

Alpha Centauri would be 140 miles away.

There would be at least 55 other star larger and smaller than our sun within 600 miles, and billions of other stars beyond that.

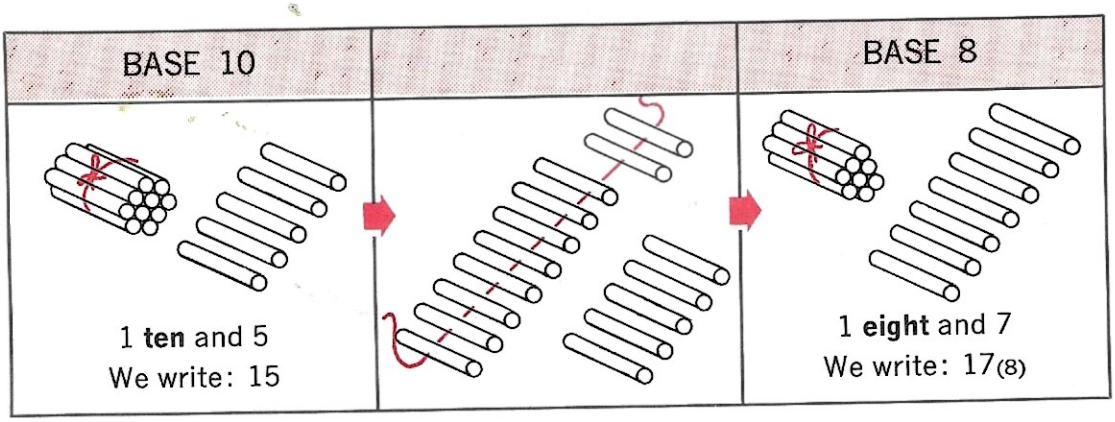
1. [A] Give the sun's diameter to the nearest hundred thousand miles.
   [B] Give the distance of the earth from the sun, rounded to the nearest 10 million miles.
   [C] Give the distance to Alpha Centauri, rounded to the nearest 10 trillion miles.

2. Exponents are used to simplify the writing of very large numbers. When a number is written as the product of a number (between 1 and 10) times a power of ten, we say that the number is written using **scientific notation**. The diameter of the earth is about 40,000,000 feet. Using scientific notation, we write 4×10^7
   Use scientific notation to write the numbers in exercise 1 and for parts 4 and 5 in the table above.

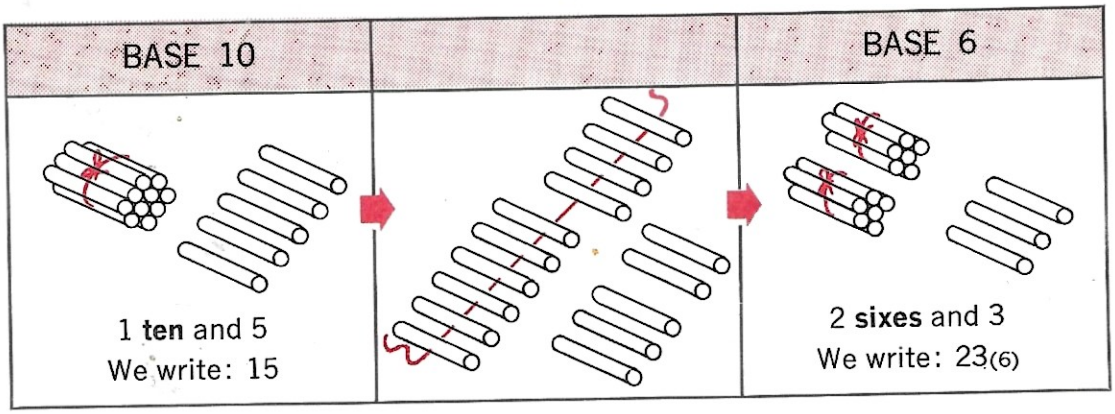
Different number bases

In our **base ten** system, we group objects by tens. The examples below show how we can use groupings other than ten to write numerals for numbers. In each example, there are 15 sticks, yet for each different grouping we write a different numeral.

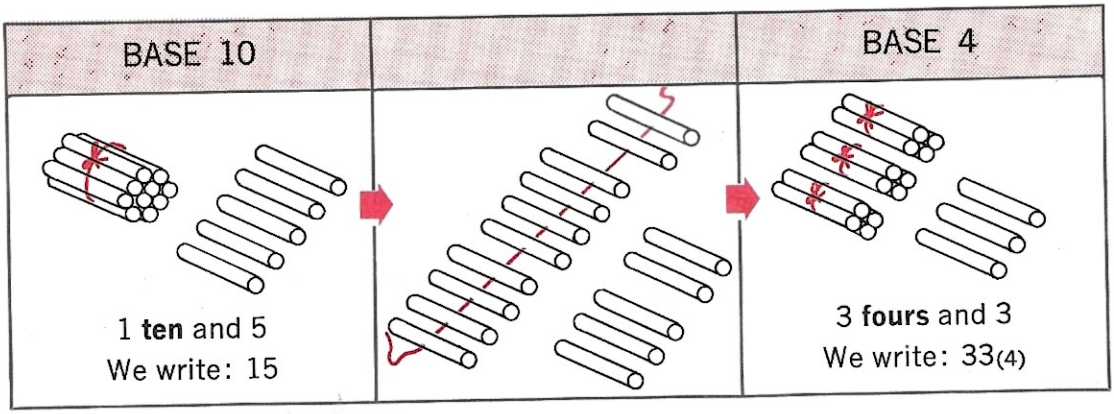
A



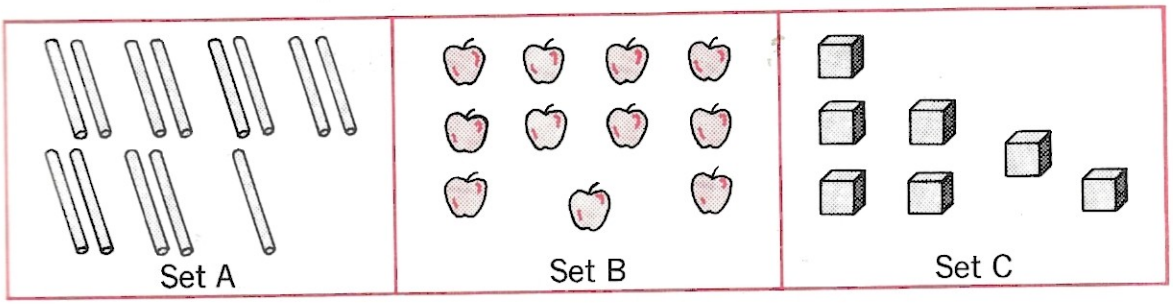
B



C



EXERCISES



1. For sets A, B, and C above, give the base ten numeral that tells how many objects are in the set.

2. [A] How many groups of 8 sticks are in set A?
   [B] After all the possible groups of 8 are made, how many extra sticks are there?
   [C] Give the base eight numeral that tells how many sticks are in set A.

3. [A] How many groups of 6 sticks are in set A?
   [B] After all the possible groups of 6 are made, how many extra sticks are there?
   [C] Give the base six numeral that tells how many sticks are in set A.

4. [A] How many groups of 4 sticks are in set A?
   [B] After all the possible groups of 4 are made, how many extra sticks are there?
   [C] Give the base four numeral that tells how many sticks are in set A.

5. Give the base eight numeral that tells how many objects are in set B; in set C.

6. Give the base six numeral that tells how many objects are in set B; in set C.

7. Give the base four numeral that tells how many objects are in set B; in set C.

★8. Give the base five numeral that tells how many objects are in set A; in set B; in set C.

★9. Give the base seven numeral that tells how many objects are in set A; in set B; in set C.

Base ten and base six

If you have 20 checkers, it is easy to place them in groups of 6 and give the base six numeral that tells how many. → $32_{(6)}$

Since there is the same number of checkers whether we group by ten or six, we write $20_{(10)} = 32_{(6)}$.

To write the base six numeral for a number such as $27_{(10)}$, you may think, "How many sixes in 27? How many left over?" Explain why you could write $27_{(10)} = 43_{(6)}$.

EXERCISES

1. For each set, write an equation that uses the base ten and base six numeral for the number of dots. Exercise 1A is completed correctly.

[A] ⁘ ⁘ ·
$13_{(10)} = 21_{(6)}$

[B] ⁘ ⁘
⁘ ⁛

[C] ⁘ ⁘ ⁛
⁘ ⁘

[D] ⁘ ⁘ ⁘
⁘ ⁘

[E] ⁘ ⁘ ⁘
⁘ ⁘ ·

[F] ⁘ ⁘ ⁘
⁘ ⁛

2. Copy and complete each equation.

[A] $16_{(10)} = \square_{(6)}$	[D] $21_{(10)} = \square_{(6)}$	[G] $30_{(10)} = \square_{(6)}$	[J] $28_{(10)} = \square_{(6)}$
[B] $25_{(10)} = \square_{(6)}$	[E] $35_{(10)} = \square_{(6)}$	[H] $5_{(10)} = \square_{(6)}$	[K] $6_{(10)} = \square_{(6)}$
[C] $20_{(10)} = \square_{(6)}$	[F] $14_{(10)} = \square_{(6)}$	[I] $4_{(10)} = \square_{(6)}$	[L] $12_{(10)} = \square_{(6)}$

3. Study the example; then complete each sentence.
(Example: $24_{(6)}$ means 2 sixes and 4 or 16 in base ten.)

[A] $35_{(6)}$ means 3 sixes and 5 or ▮ in base ten.

[B] $14_{(6)}$ means 1 six and ▮ or ▮ in base ten.

[C] $53_{(6)}$ means ▮ sixes and ▮ or ▮ in base ten.

[D] $25_{(6)}$ means ▮ sixes and ▮ or ▮ in base ten.

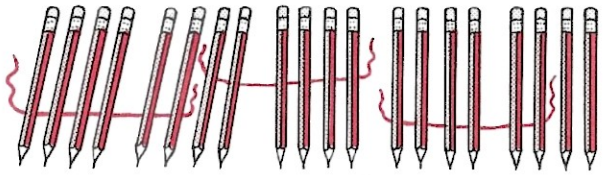
[E] $30_{(6)}$ means ▮ sixes and ▮ or ▮ in base ten.

Computing with base six numerals

Brad put 4 red pencils with 5 black pencils. Since he wanted to add using base six numerals, he put the pencils in groups of 6. He wrote

$5_{(6)} + 4_{(6)} = 13_{(6)}$

When Brad decided to multiply using base six numerals, he selected 5 sets of 4 pencils. He put the pencils in groups of 6 and wrote



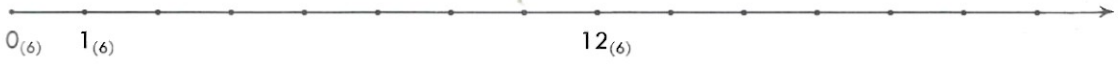
$5_{(6)} \times 4_{(6)} = 32_{(6)}$

EXERCISES

1. Find the sums. Use base six numerals.

[A] $4_{(6)} + 4_{(6)}$	[D] $5_{(6)} + 1_{(6)}$	[G] $5_{(6)} + 5_{(6)}$	[J] $1_{(6)} + 5_{(6)}$
[B] $3_{(6)} + 3_{(6)}$	[E] $3_{(6)} + 5_{(6)}$	[H] $3_{(6)} + 4_{(6)}$	[K] $4_{(6)} + 0_{(6)}$
[C] $2_{(6)} + 4_{(6)}$	[F] $3_{(6)} + 2_{(6)}$	[I] $5_{(6)} + 2_{(6)}$	[L] $4_{(6)} + 5_{(6)}$

2. Copy this number line and label it with base six numerals.



3. Find the sums. Use base six numerals.

[A] $13_{(6)}$ $+2_{(6)}$	[B] $13_{(6)}$ $+3_{(6)}$	[C] $12_{(6)}$ $+5_{(6)}$	[D] $24_{(6)}$ $+1_{(6)}$	[E] $24_{(6)}$ $+2_{(6)}$	[F] $24_{(6)}$ $+3_{(6)}$

4. Find the products. Use base six numerals.

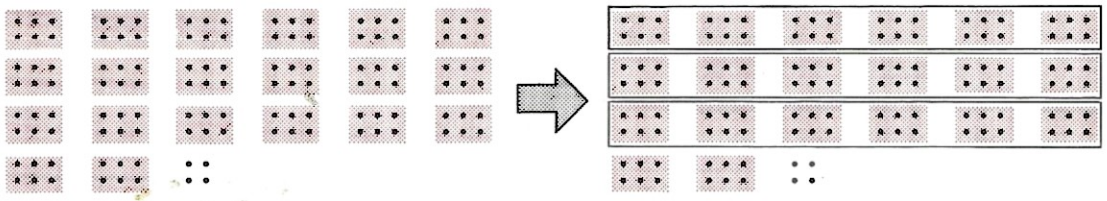
[A] $3_{(6)} \times 3_{(6)}$	[D] $4_{(6)} \times 2_{(6)}$	[G] $4_{(6)} \times 3_{(6)}$	[J] $4_{(6)} \times 4_{(6)}$
[B] $2_{(6)} \times 2_{(6)}$	[E] $2_{(6)} \times 4_{(6)}$	[H] $5_{(6)} \times 5_{(6)}$	[K] $5_{(6)} \times 1_{(6)}$
[C] $3_{(6)} \times 2_{(6)}$	[F] $3_{(6)} \times 4_{(6)}$	[I] $3_{(6)} \times 0_{(6)}$	[L] $2_{(6)} \times 3_{(6)}$

★5. Find the products. Use base six numerals.

[A] $11_{(6)}$ $\times 4_{(6)}$	[B] $12_{(6)}$ $\times 4_{(6)}$	[C] $13_{(6)}$ $\times 3_{(6)}$	[D] $21_{(6)}$ $\times 2_{(6)}$	[E] $25_{(6)}$ $\times 2_{(6)}$	[F] $30_{(6)}$ $\times 2_{(6)}$

Larger numbers—base six

Study the example to see how to write a 3-digit base six numeral that tells how many dots are in a set of dots.



First arrange the dots in groups of 6.

Then form groups of 6 sixes.

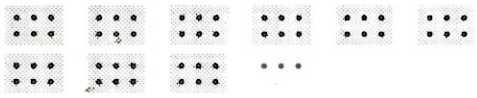
Since there are **3** groups of 6×6 (6 sixes), **2** groups of 6, and **4** extra dots, the base six numeral for the number in this set is $\mathbf{324}_{(6)}$.

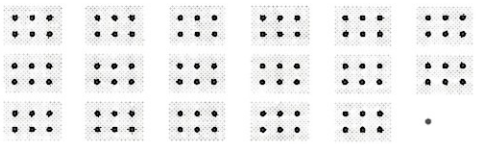
$324_{(10)}$ means $(3 \times \mathbf{10} \times \mathbf{10}) + (2 \times \mathbf{10}) + 4$.

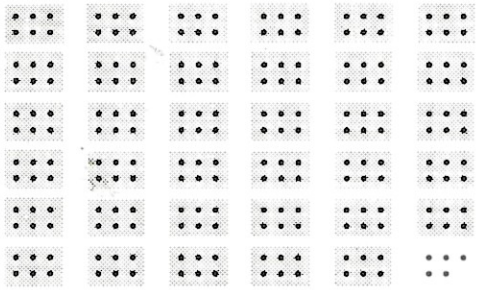
$324_{(6)}$ means $(3 \times \mathbf{6} \times \mathbf{6}) + (2 \times \mathbf{6}) + 4$.

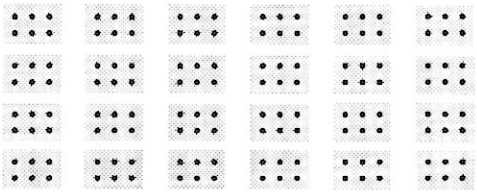
EXERCISES

1. Give the base six numeral for the number of dots in each set.

[A] 

[B] 

[C] 

[D] 

2. Copy each equation and give the missing numbers.

[A] $142_{(6)} = (\square \times 6 \times 6) + (\square \times 6) + 2$

[B] $543_{(6)} = (\square \times 6 \times 6) + (\square \times 6) + 3$

★[C] $142_{(6)} = \square_{(10)}$

★[D] $543_{(6)} = \square_{(10)}$

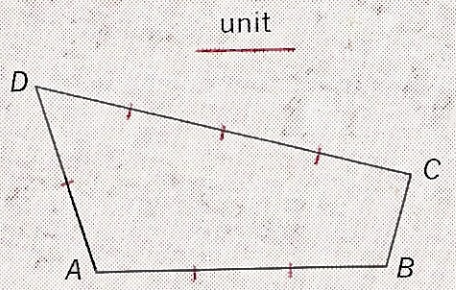
★**3.** Think about place value in base ten and decide how you would write the numeral for exercise **1c** if one more dot were included.

★**4.** Find the sum of $35_{(6)}$ and $43_{(6)}$.

Length and perimeter

- We count **segment units** to find length.
- We find the **sum** of the **lengths** of the sides to get the perimeter of a polygon.

Using the unit shown in red, we say that the length of side *AB* is 3. The perimeter of the polygon is 10.



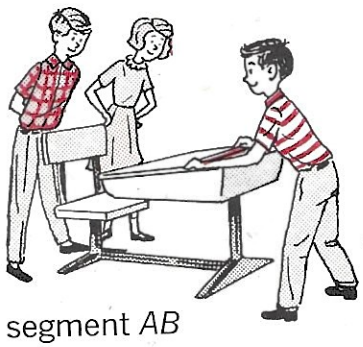
The length of an object or segment depends on the size of unit we choose. The unit used above is compared here with the inch and the centimeter units.

EXERCISES

1. [A] What is the width (side to side) of your desk to the nearest inch?

   [B] To the nearest centimeter, what is the width of your desk?

   [C] Which number is greater? Why?



2. Answer these questions without measuring.

   [A] Using the centimeter unit, would the length of segment *AB* in the figure above be greater or less than 3?

   [B] Using the inch unit, would the length of segment *AB* be greater or less than 3?

3. One kilometer (km) is a unit of length used in many countries. A kilometer is less than one mile.

   [A] According to the road sign, which is farther, Brussels or Paris?

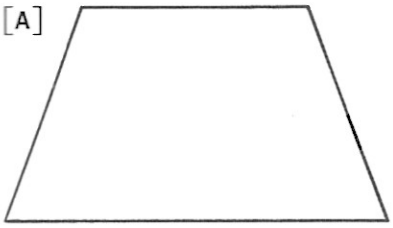
   [B] Is it more or less than 90 kilometers to Paris?

   [C] Is it more or less than 90 miles to Brussels?

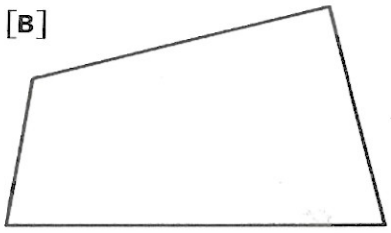
4. The mile unit usually used to measure land distances is the statute mile. The nautical mile, used at sea, is slightly greater than a statute mile. If a ship were traveling at 30 nautical miles per hour, would it go a greater or lesser distance in one hour than a ship that travels 30 statute miles per hour?

5. Use your centimeter ruler to find the perimeter of each polygon.

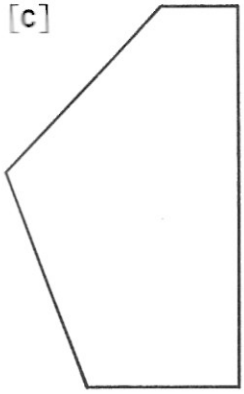
[A]



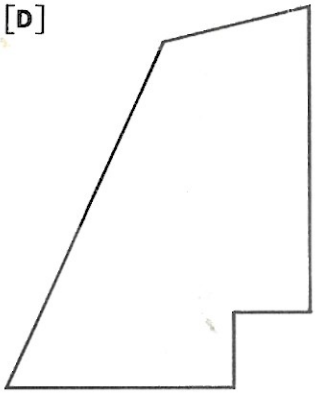
[B]



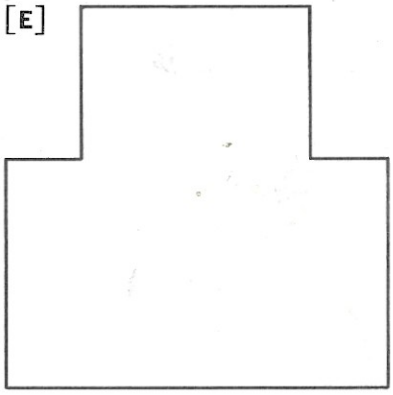
[C]



[D]



[E]



6. [A] A square has a perimeter of 36. What is the length of each side?

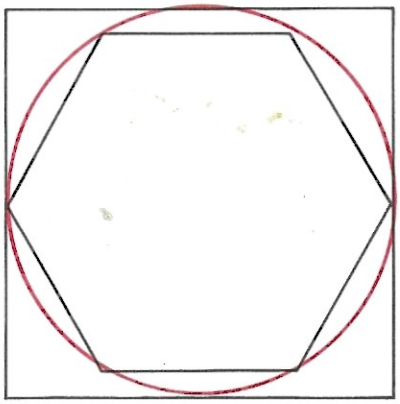
   [B] A rectangle has a perimeter of 36 and one side of 4. What is the length of each side?

   [C] A triangle has two sides that have lengths of 17 and a perimeter of 50. What is the length of the third side?

7. [A] What is the perimeter of the square in inches?

   [B] What is the perimeter of the hexagon?

   [C] The "perimeter" of the circle in inches is between what two numbers?



Addition and multiplication

Examples A and B show another way to write about our work with functions. The function rule for each example is: $f(n) = n \times 7$

A Use **3** for **n** in the *function* rule. → The output number is 3×7.

$f(3) = 21$

B Use **5** for **n** in the *function* rule. → The output number is 5×7.

$f(5) = 35$

We read "f of 3 equals 21" for $f(3) = 21$. Example C compares the work you did in the last lesson with the examples given above.

C

function rule	
$n \times 7$	
n	$f(n)$
3	21
5	35
6	42

$f(n) = n \times 7$

→ $f(3) = 21$
→ $f(5) = 35$
→ $f(6) = 42$

EXERCISES

Give the missing numbers.

1. $f(n) = n + 5$

[A] $f(7) =$ ▒
[B] $f(4) =$ ▒
[C] $f(9) =$ ▒
[D] $f(8) =$ ▒
[E] $f(6) =$ ▒

2. $f(n) = 8 \times n$

[A] $f(3) =$ ▒
[B] $f(7) =$ ▒
[C] $f(9) =$ ▒
[D] $f(8) =$ ▒
[E] $f(6) =$ ▒

3. $f(n) = n \times 9$

[A] $f(5) =$ ▒
[B] $f(9) =$ ▒
[C] $f(7) =$ ▒
[D] $f(6) =$ ▒
[E] $f(8) =$ ▒

4. For the rule $f(n) = 7 \times n$, we know that $f(8) =$ ▒.

5. For the rule $f(n) = n + 7$, we know that $f(8) =$ ▒.

6. For the rule $f(n) = n \times 5$, we know that $f(8) =$ ▒.

7. For the rule $f(n) = 4 + n$, we know that $f(8) =$ ▒.

8. $f(n) = (2 \times n) + 1 \longrightarrow f(8) =$ ▒

9. $f(n) = (3 \times n) + 2 \longrightarrow f(8) =$ ▒

10. $f(n) = 6 + (2 \times n) \longrightarrow f(6) =$ ▒

11. $f(n) = 4 \times (3 + n) \longrightarrow f(5) =$ ▒

12. $f(n) = 5 \times (n + 6) \longrightarrow f(2) =$ ▒

13. $f(n) = (6 \times n) + 7 \longrightarrow f(3) =$ ▒

14. $f(n) = 9 \times (n + 1) \longrightarrow f(0) =$ ▒

15. $f(n) = n \times (n + 1) \longrightarrow f(9) =$ ▒

Addition and multiplication are related. The examples below help describe this relation.

A $f(n) = 3 \times n$

$f(6) = 18$

$f(4) = 12$

$f(9) = 27$

⋮ ⋮

B $f(n) = n + n + n$

$f(6) = 18$

$f(4) = 12$

$f(9) = 27$

⋮ ⋮

Notice that when the input numbers are the same, these two function rules give the same output numbers.

DISCUSSION EXERCISES

1. What is $f(5)$ for rule A above?
2. What is $f(5)$ for rule B above?
3. What is $f(8)$ for rule A above?
4. What is $f(8)$ for rule B above?
5. Study these two function rules.

$f(n) = 2 \times n$ $\quad$ $f(n) = n + n$

[A] What is $f(5)$ for the first rule?

[B] What is $f(5)$ for the second rule?

[C] What is $f(8)$ for the first rule?

[D] What is $f(8)$ for the second rule?

EXERCISES

1. Give the missing numbers.

$f(n) = 4 \times n$

[A] $f(2) =$ ▒▒

[C] $f(3) =$ ▒▒

[E] $f(4) =$ ▒▒

$f(n) = n + n + n + n$

[B] $f(2) =$ ▒▒

[D] $f(3) =$ ▒▒

[F] $f(4) =$ ▒▒

2. What is the "addition rule" that corresponds to:

[A] $f(n) = 5 \times n$ $\quad$ [B] $f(n) = 9 \times n$ $\quad$ [C] $f(n) = 6 \times n$

3. What is the "multiplication rule" that corresponds to:

[A] $f(n) = n + n + n + n + n + n + n$

[B] $f(n) = n + n + n + n + n$

[C] $f(n) = n + n + n + n + n + n + n + n$

Subtraction and addition

We find differences by thinking about missing addends. The diagram shows that the difference **14 − 5** is the addend that adds to **5** and gives the sum **14**.

You have found this difference

A A S
$n + 5 = 14$

S A A
$14 - 5 = n$

when you find this addend.

EXERCISES

1. Find the missing addend in the addition equation. Then copy the subtraction equation, giving the correct difference.

[A] $n + 8 = 14$
$14 - 8 = n$

[B] $a + 9 = 15$
$15 - 9 = a$

[C] $c + 3 = 11$
$11 - 3 = c$

[D] $b + 4 = 12$
$12 - 4 = b$

[E] $r + 9 = 17$
$17 - 9 = r$

[F] $t + 8 = 16$
$16 - 8 = t$

[G] $x + 7 = 13$
$13 - 7 = x$

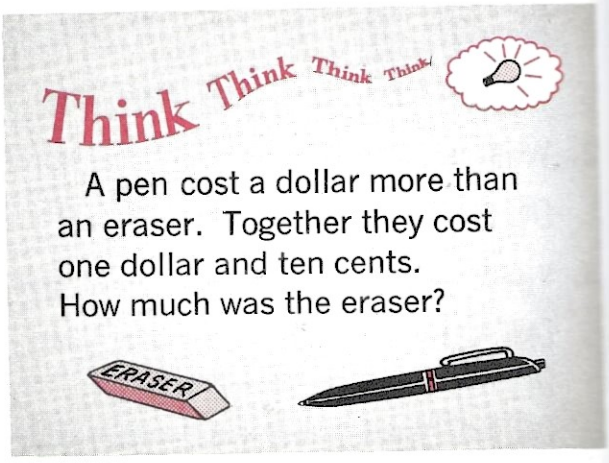
[H] $s + 5 = 14$
$14 - 5 = s$

2. Solve the equations.

[A] $6 + 7 = a$
[B] $i + 8 = 13$
[C] $15 = 10 + n$
[D] $12 - 4 = r$
[E] $16 - 8 = f$
[F] $11 - n = 6$
[G] $3 + q = 12$
[H] $7 + 8 = p$
[I] $14 = 9 + b$
[J] $17 - q = 9$
[K] $s - 7 = 3$
[L] $11 = t + 9$
[M] $i = 8 + 7$
[N] $13 - k = 7$
[O] $5 + x = 10$
[P] $e - 4 = 10$
[Q] $13 - d = 10$
[R] $12 - 6 = v$
[S] $c = 10 - 1$
[T] $p + 7 = 17$
[U] $13 - y = 5$
[V] $n - 8 = 7$
[W] $14 - 5 = m$
[X] $w + 10 = 16$

3. Solve the equations.

[A] $(36 - 9) + 9 = n$
[B] $(48 - 19) + 19 = c$
[C] $(75 - 46) + b = 75$
[D] $(32 - a) + 15 = 32$
[E] $(s - 68) + 68 = 75$
[F] $(18 - 7) + 7 = r$
[G] $(12 - 7) + t = 12$
[H] $(63 - a) + 19 = 63$


Think Think Think Think
A pen cost a dollar more than an eraser. Together they cost one dollar and ten cents. How much was the eraser?
ERASER

Division and multiplication

We find quotients by thinking about missing factors. The diagram shows that the quotient $28 \div 4$ is the factor that multiplies by **4** and gives the product **28**.

You have found this quotient $28 \div 4 = \mathbf{n}$ (P F F)

$\mathbf{n} \times 4 = 28$ (F F P)

when you find this factor.

EXERCISES

1. Find the missing factor in the multiplication equation. Then copy the division equation, giving the correct quotient.

[A] $\mathbf{n} \times 5 = 30$ $30 \div 5 = \mathbf{n}$	[C] $\mathbf{c} \times 7 = 42$ $42 \div 7 = \mathbf{c}$	[E] $\mathbf{r} \times 9 = 63$ $63 \div 9 = \mathbf{r}$	[G] $\mathbf{x} \times 6 = 54$ $54 \div 6 = \mathbf{x}$
[B] $\mathbf{a} \times 4 = 28$ $28 \div 4 = \mathbf{a}$	[D] $\mathbf{b} \times 8 = 40$ $40 \div 8 = \mathbf{b}$	[F] $\mathbf{t} \times 4 = 32$ $32 \div 4 = \mathbf{t}$	[H] $\mathbf{s} \times 9 = 0$ $0 \div 9 = \mathbf{s}$

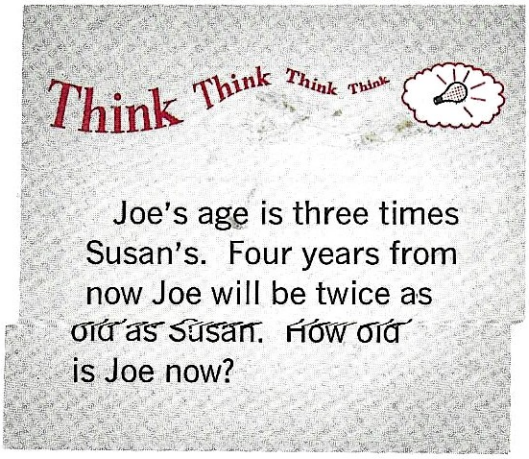
2. Solve the equations.

[A] $\mathbf{p} \times 7 = 21$	[G] $5 \times \mathbf{b} = 25$	[M] $5 \times \mathbf{e} = 45$	[S] $(54 \div 6) \times 6 = \mathbf{n}$
[B] $35 \div 7 = \mathbf{k}$	[H] $56 \div 7 = \mathbf{h}$	[N] $7 \times 6 = \mathbf{x}$	[T] $(72 \div 9) \times 9 = \mathbf{c}$
[C] $7 \times \mathbf{s} = 56$	[I] $42 \div 7 = \mathbf{m}$	[O] $32 \div \mathbf{f} = 4$	[U] $(56 \div 8) \times \mathbf{b} = 56$
[D] $\mathbf{g} \times 4 = 24$	[J] $56 \div \mathbf{j} = 7$	[P] $72 \div \mathbf{y} = 8$	[V] $(32 \div \mathbf{a}) \times 8 = 32$
[E] $42 \div 6 = \mathbf{n}$	[K] $54 = 6 \times \mathbf{n}$	[Q] $\mathbf{a} \times 7 = 49$	[W] $(63 \div \mathbf{n}) \times 9 = 63$
[F] $9 \times \mathbf{c} = 18$	[L] $8 \times \mathbf{i} = 40$	[R] $40 \div 8 = \mathbf{n}$	[X] $(\mathbf{n} \div 7) \times 7 = 49$

3. [A] Does the equation $\mathbf{n} \times 0 = 7$ (F F P) have a solution?

[B] Does the equation $\mathbf{n} \times 0 = 0$ (F F P) have **just one** solution?

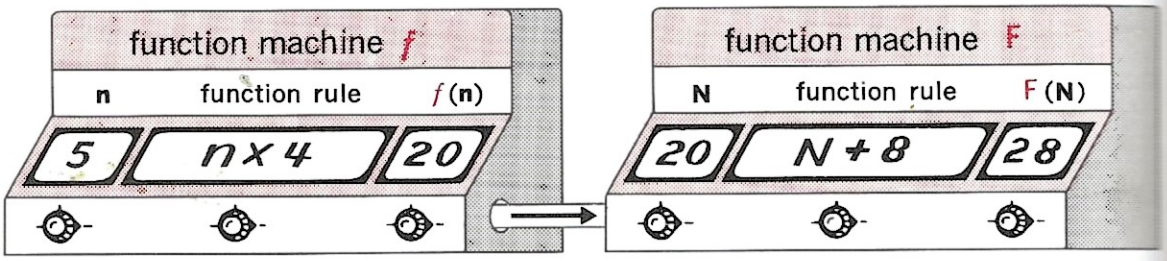
[C] Since $7 \div 0 = \mathbf{n}$ (P F F) has no solution and $0 \div 0 = \mathbf{n}$ (F F F) has many solutions, we never divide by ▓.



Joe's age is three times Susan's. Four years from now Joe will be twice as old as Susan. How old is Joe now?

Combining operations

Study the figure below to see how we use two function machines together.



Notice that the output for machine f is the input for machine F. Study this table and the function machines to see how we find the output number for F when we know the input number for f.

RULE	$f(n) = n \times 4$
	$F(N) = N + 8$

n	F(N)
5	28
3	20
6	32

EXERCISES

For each exercise, think about connected function machines f and F, as in the example above. Remember that the output number for machine f is the input number for machine F. Then give the missing numbers. The answers for part A of exercises 1, 2, and 3 are given.

1.

RULE	$f(n) = n \times 3$
	$F(N) = N + 5$

	n	F(N)
[A]	4	17
[B]	3	
[C]	1	
[D]	5	
[E]	2	
[F]	6	

2.

RULE	$f(n) = n + 6$
	$F(N) = N \div 3$

	n	F(N)
[A]	9	5
[B]	12	
[C]	15	
[D]	6	
[E]	0	
[F]	21	

3.

RULE	$f(n) = n - 7$
	$F(N) = N \times 6$

	n	F(N)
[A]	12	30
[B]	13	
[C]	16	
[D]	8	
[E]	10	
[F]	7	

4.

RULE: $f(n) = n - 5$; $F(N) = N \div 6$

	n	F(N)
[A]	35	
[B]	17	
[C]	23	
[D]	65	
[E]	47	
[F]	29	

5.

RULE: $f(n) = n \times 8$; $F(N) = N \div 4$

	n	F(N)
[A]	2	
[B]	3	
[C]	1	
[D]	5	
[E]	4	
[F]	0	

6.

RULE: $f(n) = n - n$; $F(N) = N \times 9$

	n	F(N)
[A]	6	
[B]	5	
[C]	7	
[D]	9	
[E]	3	
[F]	8	

7.

RULE: $f(n) = n + n$; $F(N) = N \div 2$

	n	F(N)
[A]	4	
[B]	6	
[C]	9	
[D]	8	
[E]	7	
[F]	3	

8.

RULE: $f(n) = n + 8$; $F(N) = N - 8$

	n	F(N)
[A]	6	
[B]	5	
[C]	7	
[D]	3	
[E]	0	
[F]	9	

9.

RULE: $f(n) = n \times 6$; $F(N) = N \div 6$

	n	F(N)
[A]	7	
[B]	5	
[C]	9	
[D]	6	
[E]	8	
[F]	3	

10.

RULE: $f(n) = n + 7$; $F(N) = N \times 0$

	n	F(N)
[A]	9	
[B]	7	
[C]	6	
[D]	1	
[E]	8	
[F]	4	

11.

RULE: $f(n) = n + 0$; $F(N) = N - 0$

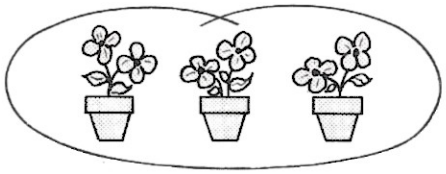
	n	F(N)
[A]	6	
[B]	5	
[C]	0	
[D]	7	
[E]	1	
[F]	9	

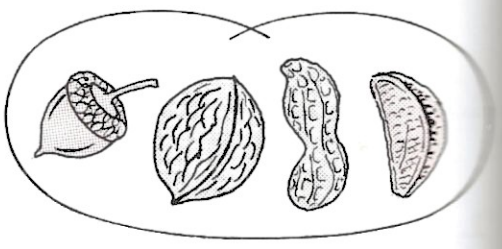
12.

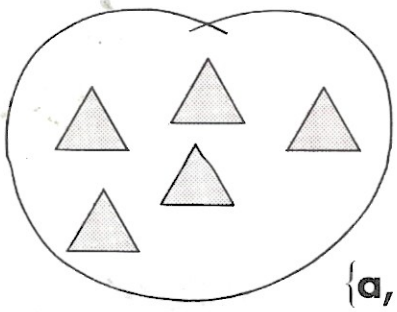
RULE: $f(n) = n \div n$; $F(N) = N \times N$

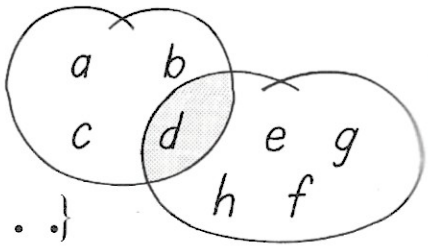
	n	F(N)
[A]	8	
[B]	5	
[C]	7	
[D]	2	
[E]	6	
[F]	3	

Short stories—sets









{a, b, c, d, . . .}

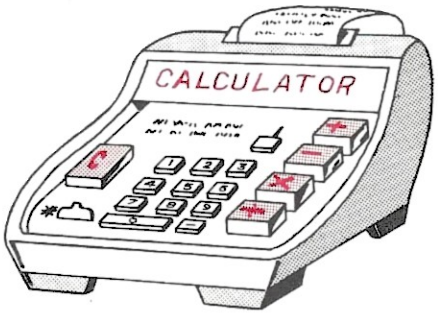
1. Six sets of 7. One set of 8. How many in all?
2. One set of 35 and one set of 7. How many more in these two sets together than in one set of 9?
3. How many more in six sets of 4 than in one set of 8?
4. Nine sets of 2. Six sets of 5. How many in all?
5. One set of 50. Remove a set of 2. Arrange the rest into 6 equivalent sets (same number in each). How many in each set?
6. Arrange a set of 35 into 5 equivalent sets. How many more in one set of 10 than in one of these 5 sets?
7. One set of 39. One set of 6. Arrange these two sets together into 9 equivalent sets. How many in each set?
8. How many more in seven sets of 8 than in seven sets of 3?
9. Arrange a set of 63 into 7 equivalent sets. How many in 4 of these sets?
10. Six sets of 6. Take these 6 sets together and arrange them into 4 equivalent sets. How many in each?

★ **11.** Eight sets of 9. Remove 2 from each set. Now how many in the 8 sets together?

★ **12.** Arrange a set of 54 into 6 equivalent sets. How many in a set of 25 together with three of the 6 sets above?

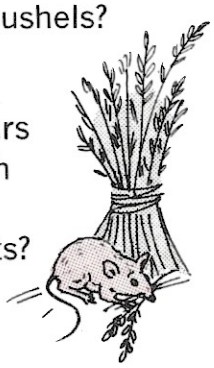
Add, subtract, multiply, or divide

No numbers are given in these short-story problems. Use **A**, **S**, **M**, or **D** to tell which operation (**A**ddition, **S**ubtraction, **M**ultiplication, or **D**ivision) you would use to solve each problem. For a problem where two operations are needed, say subtraction and division, you should answer **S**, **D**.



1. Corn: ▒ bushels per acre. ▒ acres. How many bushels?

2. Wheat: ▒ dollars per bushel. Oats: ▒ dollars per bushel. How much more per bushel is the wheat than the oats?



3. Bought a bike for ▒. Sold it for ▒. Sold it for how much less?

4. ▒ cookies in each box. ▒ cookies in all. How many boxes of cookies?

5. Triangle sides: ▒ ft, ▒ ft, and ▒ ft. What is the perimeter?

6. Rectangle: length ▒ in., width ▒ in. What is the area?

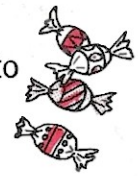
7. Went ▒ miles. Then went ▒ more miles. How far?

8. Bought ▒ yards of material for ▒ cents. How much for one yard?

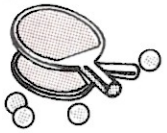
9. ▒ pads of paper. ▒ sheets on each pad. Used ▒ sheets. How many sheets left?

10. Jim had ▒ cents. Tom had ▒ times as much as Jim. How much did they have together?

11. ▒ children get ▒ pieces of candy. Same number to each. Joe gave away ▒ of his. How much did he have left?



★12. Had ▒ cents. Spent ▒ cents. Then spent all the rest on ▒ ping-pong balls. How much per ball?



★13. Had ▒ stamps. Got ▒ more stamps. *Put them in* stamp book. Same number on each of ▒ pages. How many per page?

★14. ▒ gallons and ▒ quarts of milk. How many pints of milk?

Basic principles for whole numbers

The basic principles for addition and multiplication of whole numbers and important uses of these principles are reviewed below.

ADDITION

0 Principle When you choose a whole number and add 0, the sum is the number you chose.

Examples: $5+0=5$ $\quad 987+0=987$

Commutative principle
You can change the order of the addends and the sum is the same. Examples:

$5+7=7+5$
$863+92=92+863$

Associative principle
You can change the grouping of the addends and the sum is the same. Examples:

$(2+7)+4=2+(7+4)$
$(18+68)+51=18+(68+51)$

Since you can change both the order and grouping of the addends, you can arrange addends any way that is convenient.

Examples:

$8+7+2+3=8+2+7+3$ $\quad 27+58+73=73+27+58$

MULTIPLICATION

1 Principle When you choose a whole number and multiply by 1, the product is the number you chose.

Examples: $9\times1=9$ $\quad 834\times1=834$

Commutative principle
You can change the order of the factors and the product is the same. Examples:

$5\times7=7\times5$
$863\times92=92\times863$

Associative principle
You can change the grouping of the factors and the product is the same. Examples:

$(2\times7)\times4=2\times(7\times4)$
$(18\times68)\times51=18\times(68\times51)$

Since you can change both the order and grouping of the factors, you can arrange factors any way that is convenient.

Examples:

$6\times5\times7\times2=6\times7\times5\times2$ $\quad 25\times38\times4\times10=25\times4\times10\times38$

MULTIPLICATION and ADDITION

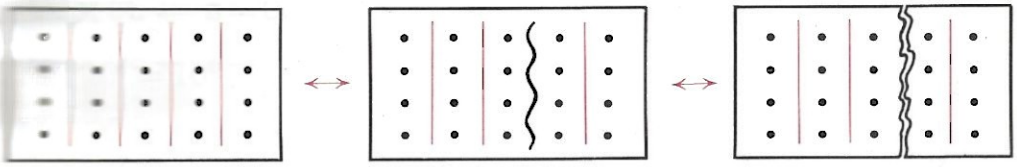
Distributive principle

The distributive principle can be described as follows:

You can "break apart" either factor when you multiply.

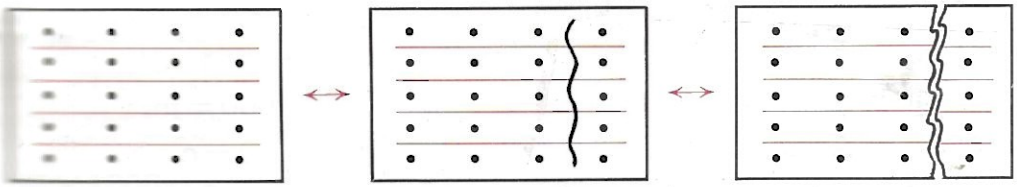
This principle can be illustrated best by using sets. Example A illustrates "breaking apart" the first factor. Example B illustrates "breaking apart" the second factor.

A



$5 \times 4 = (3+2) \times 4 = (3 \times 4) + (2 \times 4)$

B



$5 \times 4 = 5 \times (3+1) = (5 \times 3) + (5 \times 1)$

EXERCISES

1. Solve the equations.

[A] $68 + 27 = n + 68$

[B] $(84 + 26) + 13 = 84 + (n + 13)$

[C] $74 \times n = 61 \times 74$

[D] $(n \times 32) \times 15 = 89 \times (32 \times 15)$

[E] $6 \times 13 = 6 \times (n + 6)$

[F] $84 + 63 + 27 = 63 + n + 84$

[G] $9 \times 56 \times 91 = 91 \times 9 \times n$

[H] $8 \times 7 = (n + 2) \times 7$

[I] $18 \times 37 \times 2 = n \times 37 \times 18$

[J] $64 + 23 + 13 = 23 + 64 + n$

2. Solve the equations.

[A] $5 \times 12 = n \times (10 + 2)$

[B] $6 \times 74 = 6 \times (n + 4)$

[C] $8 \times 21 = 8 \times (20 + n)$

[D] $7 \times 56 = 7 \times (n + 6)$

[E] $2 \times 98 = n \times (90 + 8)$

[F] $5 \times 12 = (5 \times n) + (5 \times 2)$

[G] $6 \times 74 = (n \times 70) + (6 \times 4)$

[H] $8 \times 21 = (8 \times 20) + (8 \times n)$

[I] $7 \times 56 = (7 \times 50) + (n \times 6)$

[J] $2 \times 98 = (n \times 90) + (n \times 8)$

Special products and quotients

EXERCISES

1. Give the number for **a**; then give the number for **b**.

[A] $15 \times 10 = a \rightarrow 3 \times 50 = b$

[B] $42 \times 10 = a \rightarrow 6 \times 70 = b$

[C] $24 \times 10 = a \rightarrow 4 \times 60 = b$

[D] $18 \times 100 = a \rightarrow 3 \times 600 = b$

[E] $45 \times 100 = a \rightarrow 5 \times 900 = b$

[F] $28 \times 100 = a \rightarrow 4 \times 700 = b$

[G] $35 \times 100 = a \rightarrow 50 \times 70 = b$

[H] $48 \times 100 = a \rightarrow 60 \times 80 = b$

[I] $27 \times 100 = a \rightarrow 30 \times 90 = b$

[J] $36 \times 100 = a \rightarrow 90 \times 40 = b$

2. Find the quotients.

[A] Since $6 \times 40 = 240$, we know that $240 \div 6 = a$.

[B] Since $5 \times 90 = 450$, we know that $450 \div 90 = r$.

[C] Since $7 \times 800 = 5600$, we know that $5600 \div 7 = x$.

[D] Since $9 \times 600 = 5400$, we know that $5400 \div 600 = m$.

[E] Since $80 \times 30 = 2400$, we know that $2400 \div 80 = c$.

[F] Since $30 \times 40 = 1200$, we know that $1200 \div 40 = n$.

[G] Since $70 \times 20 = 1400$, we know that $1400 \div 20 = t$.

[H] Since $60 \times 90 = 5400$, we know that $5400 \div 60 = q$.

[I] Since $90 \times 30 = 2700$, we know that $2700 \div 90 = b$.

[J] Since $60 \times 50 = 3000$, we know that $3000 \div 50 = a$.

[K] Since $70 \times 40 = 2800$, we know that $2800 \div 40 = y$.

[L] Since $50 \times 90 = 4500$, we know that $4500 \div 50 = s$.

3. Find the products and quotients.

[A] 7×60

[B] $180 \div 20$

[C] 80×20

[D] $240 \div 40$

[E] $2400 \div 80$

[F] 70×70

[G] $3600 \div 60$

[H] 50×90

[I] $4200 \div 60$

[J] 30×90

[K] $3500 \div 50$

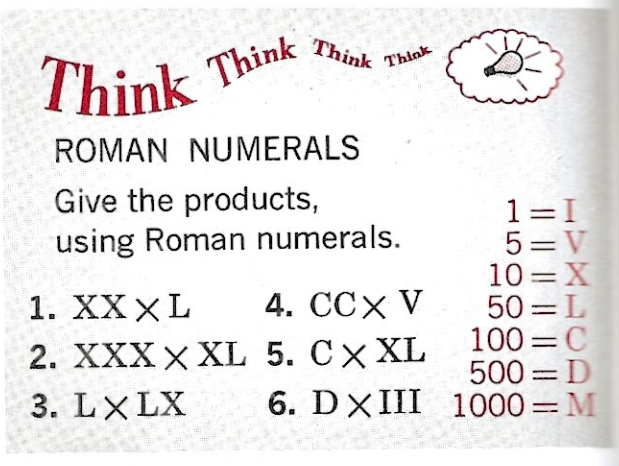
[L] $4500 \div 90$

[M] 5×900

[N] 80×30

[O] $1800 \div 30$

[P] 70×80



4. Finding the product $(7 \times 10^1) \times (6 \times 10^2)$ is much like finding the product 70×600. The exponents help you "keep track" of the zeros. Study the examples.

(A) $70 \times 600 = 42{,}000$

$(7 \times 10^1) \times (6 \times 10^2) = 42 \times 10^3$

(B) $800 \times 3000 = 2{,}400{,}000$

$(8 \times 10^2) \times (3 \times 10^3) = 24 \times 10^5$

In each exercise, give the numbers for a and b.

[A] $10 \times 10 = a$
$10^1 \times 10^1 = 10^b$

[B] $10 \times 100 = a$
$10^1 \times 10^2 = 10^b$

[C] $10 \times 1000 = a$
$10^1 \times 10^3 = 10^b$

[D] $100 \times 100 = a$
$10^2 \times 10^2 = 10^b$

[E] $100 \times 1000 = a$
$10^2 \times 10^3 = 10^b$

[F] $50 \times 40 = a$
$(5 \times 10^1) \times (4 \times 10^1) = b \times 10^2$

[G] $50 \times 700 = a$
$(5 \times 10^1) \times (7 \times 10^2) = b \times 10^3$

[H] $300 \times 40 = a$
$(3 \times 10^2) \times (4 \times 10^1) = 12 \times 10^b$

[I] $30 \times 8000 = a$
$(3 \times 10^1) \times (8 \times 10^3) = 24 \times 10^b$

[J] $700 \times 900 = a$
$(7 \times 10^2) \times (9 \times 10^2) = 63 \times 10^b$

[K] $200 \times 6000 = a$
$(2 \times 10^2) \times (6 \times 10^3) = 12 \times 10^b$

[L] $3000 \times 500 = a$
$(3 \times 10^3) \times (5 \times 10^2) = 15 \times 10^b$

5. Find the products.

[A] 10×10

[B] 100×10

[C] $10^3 \times 10^1$

[D] 10×100

[E] $10^2 \times 10^2$

[F] $10^3 \times 10^2$

[G] 50×70

[H] 800×60

[I] 3000×20

[J] 40×900

[K] 600×700

[L] 5000×900

6. Find the quotients.

[A] $27{,}000 \div 90$

[B] $56{,}000 \div 80$

[C] $320{,}000 \div 40$

[D] $210{,}000 \div 300$

Think Think Think Think

Give the correct sign (< or >) for each ▢.

1. 100×10^9 ▢ 10^{12}
2. 10^{100} ▢ 100×10^{10}
3. $10^{10} \times 10^{10}$ ▢ 10^{100}
4. 10^{100} ▢ 100^{10}

Finding products

The distributive principle helps you find products like 3×47. Study the example to see how this is done. You may be able to do this without using pencil and paper.

$3 \times 40 = 120$
$3 \times 7 = 21$
$\rightarrow 3 \times 47 = 141$

EXERCISES

1. Find the numbers for **a** and **b**. Then find the product for **c**.

[A] $4 \times 20 = \mathbf{a}$, $4 \times 1 = \mathbf{b}$ $\rightarrow 4 \times 21 = \mathbf{c}$

[B] $5 \times 30 = \mathbf{a}$, $5 \times 5 = \mathbf{b}$ $\rightarrow 5 \times 35 = \mathbf{c}$

[C] $6 \times 40 = \mathbf{a}$, $6 \times 3 = \mathbf{b}$ $\rightarrow 6 \times 43 = \mathbf{c}$

[D] $7 \times 60 = \mathbf{a}$, $7 \times 3 = \mathbf{b}$ $\rightarrow 7 \times 63 = \mathbf{c}$

[E] $4 \times 90 = \mathbf{a}$, $4 \times 7 = \mathbf{b}$ $\rightarrow 4 \times 97 = \mathbf{c}$

[F] $5 \times 80 = \mathbf{a}$, $5 \times 3 = \mathbf{b}$ $\rightarrow 5 \times 83 = \mathbf{c}$

2. Find the products. Write only the answers.

[A] 3×21	[E] 7×21	[I] 7×32	[M] 3×18	[Q] 9×22
[B] 6×14	[F] 4×36	[J] 5×36	[N] 4×41	[R] 4×43
[C] 5×32	[G] 3×45	[K] 4×24	[O] 8×32	[S] 6×27
[D] 6×12	[H] 6×24	[L] 2×76	[P] 6×72	[T] 7×36

3. Find the product for **a**; then find the product for **b**. Write only the answers.

[A] $5 \times 32 = \mathbf{a} \rightarrow 50 \times 32 = \mathbf{b}$

[B] $6 \times 24 = \mathbf{a} \rightarrow 60 \times 24 = \mathbf{b}$

[C] $7 \times 32 = \mathbf{a} \rightarrow 7 \times 320 = \mathbf{b}$

[D] $6 \times 72 = \mathbf{a} \rightarrow 60 \times 720 = \mathbf{b}$

[E] $3 \times 45 = \mathbf{a} \rightarrow 30 \times 450 = \mathbf{b}$

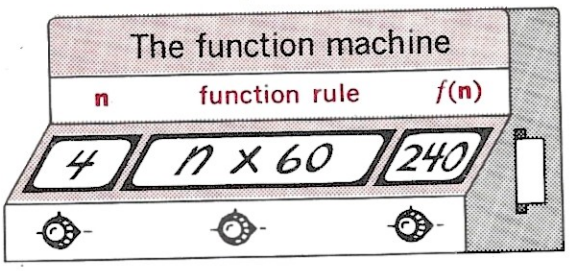
[F] $8 \times 32 = \mathbf{a} \rightarrow 800 \times 320 = \mathbf{b}$

4. Find the products. Write only the answers.

[A] 40×36	[C] 50×36	[E] 20×76	[G] 40×41	[I] 60×12
[B] 70×21	[D] 30×21	[F] 70×36	[H] 90×22	[J] 60×27

Number pairs

This function machine prints an input-output card each time it operates. The machine prints the card shown below for the numbers and rule shown on the machine.



n	f(n)
4	240

EXERCISES

1. These cards are from the machine above. Give the missing numbers.

[A]

n	f(n)
7	[redacted]

[B]

n	f(n)
[redacted]	120

[C]

n	f(n)
90	[redacted]

[D]

n	f(n)
[redacted]	4800

2. A function rule is given for each row of cards. Give the missing numbers.

$f(n) = n \times 30$

[A]

n	f(n)
80	[redacted]

[B]

n	f(n)
[redacted]	180

[C]

n	f(n)
600	[redacted]

$f(n) = 800 \times n$

[D]

n	f(n)
70	[redacted]

[E]

n	f(n)
[redacted]	1600

[F]

n	f(n)
900	[redacted]

$f(n) = n \times 7 \times 10^2$

[G]

n	f(n)
5	[redacted]

[H]

n	f(n)
10	[redacted]

[I]

n	f(n)
60	[redacted]

$f(n) = 5 \times 10^3 \times n$

[J]

n	f(n)
9	[redacted]

[K]

n	f(n)
[redacted]	150,000

[L]

n	f(n)
70	[redacted]

*3. Make a set of six cards for each of the following function rules.

[A] $20 \times n$ [B] $500 \times n$ [C] $6 \times 10^2 \times n$ [D] $3 \times 10^3 \times n$

*4. Give a function rule for this set of cards.

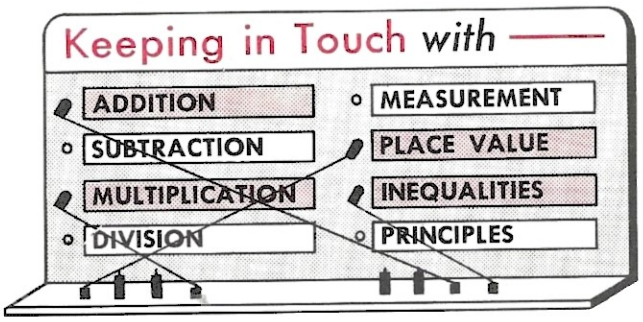
n	f(n)
2	1400

n	f(n)
4	2800

n	f(n)
0	0

n	f(n)
1	700

n	f(n)
9	6300



1. Write each number using expanded notation.

[A] 826
[B] 2745
[C] 1032
[D] 52,280
[E] 34,576
[F] 80,365
[G] 469,506
[H] 9,037,260

2. Give the correct sign ($<$, $>$, or $=$) for each ▢.

[A] $78{,}263$ ▢ $70{,}000 + (8 \times 1000) + (2 \times 10^2) + 60 + 3$

[B] $5{,}293{,}648$ ▢ 5293×10^2

[C] $387 + 256$ ▢ $387 + 246$

[D] $792 + 604$ ▢ $782 + 604$

[E] $9203 + 6152$ ▢ $9203 + 6252$

[F] $6327 + 5461$ ▢ $6337 + 5461$

[G] $72{,}834 + 659$ ▢ $73{,}834 + 965$

3. Find the products.

[A] $10^1 \times 10^2$ [B] $10^2 \times 10^2$ [C] $10^2 \times 10^3$ [D] $10^3 \times 10^4$

4. Write each number using expanded notation (exponent form).

[A] 328 [B] 7368 [C] 87,654

5. Write the ordinary base ten numeral for each exercise.

[A] 7×10^2 [C] 6×10^3 [E] 38×10^5 [G] 60×10^2 [I] 600×10^4

[B] 3×10^4 [D] 25×10^1 [F] 43×10^4 [H] 160×10^3 [J] 750×10^4

6. Round these population figures (for a recent year) to the nearest million.

[A] Canada: 18,238,247 [B] Mexico: 36,091,000

7. Give the missing numbers.

Power of ten	Ordinary base ten numeral
10^4	[A]
[B]	1000
10^5	[C]
10^7	[D]
[E]	100,000,000

Think Think Think Think

These counting sequences are in base six. Give the missing numerals.

1. 3, 4, ▢, ▢, ▢, 12, 13
2. 34, 33, 32, ▢, ▢, ▢, 24
3. 424, 425, ▢, ▢, ▢, 433
4. 52, 53, ▢, ▢, ▢, 101, 102

Chapter review

1. Solve the equations.

[A] $8+7=a$ [D] $24\div 6=t$ [G] $17-b=9$ [J] $6\times s=48$
[B] $6+n=13$ [E] $15-7=d$ [H] $8\times 9=m$ [K] $g+7=14$
[C] $9\times 3=c$ [F] $r\times 6=42$ [I] $42\div 7=x$ [L] $48\div 6=q$

2. List the missing numbers for each table.

	function rule $n\times 40$	
	n	$f(n)$
[A]	60	
[B]		1200
[C]	80	
[D]		360

	function rule $n\times 70$	
	n	$f(n)$
[E]	10	
[F]	4	
[G]	30	
[H]		2800

	function rule $n\div 60$	
	n	$f(n)$
[I]	1800	
[J]	4800	
[K]		20
[L]	6000	

3. Function rule: $f(n)=(7\times n)+8$. Give the number for:

[A] $f(3)$ [B] $f(6)$ [C] $f(20)$ [D] $f(50)$ [E] $f(10)$ [F] $f(80)$

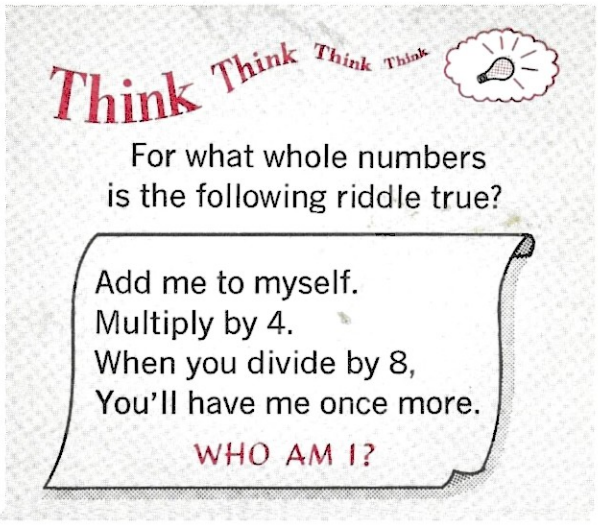
4. Find the numbers for **a**, **b**, and **c**.

[A] $5\times 70=a$, $5\times 3=b$ $\longrightarrow 5\times 73=c$

[B] $9\times 40=a$, $9\times 2=b$ $\longrightarrow 9\times 42=c$

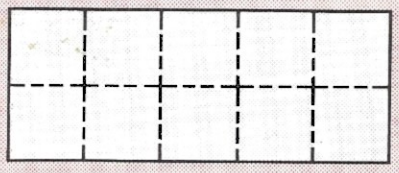
5. List the number pair for each row.

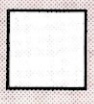
	Products	×	Differences
[A]	24		5
[B]	56		1
[C]	35		2
[D]	64		0
[E]	80		2


Think Think Think Think
For what whole numbers
is the following riddle true?
Add me to myself.
Multiply by 4.
When you divide by 8,
You'll have me once more.
WHO AM I?

Area and surface area

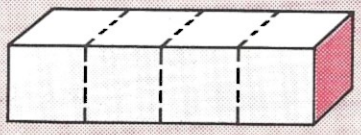
▶ We count square units to find the **area** of plane figures.





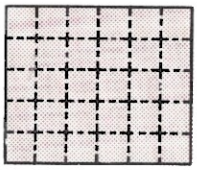
unit

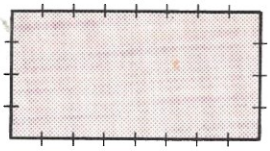
▶ We count square units to find the **surface area** of space figures.

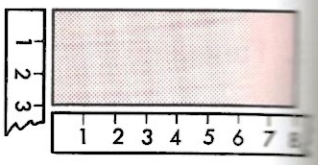


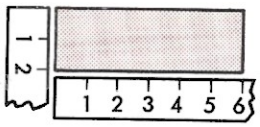
EXERCISES

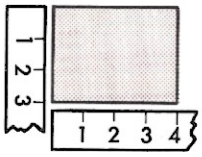
1. Give the area for each figure. Use the units given.

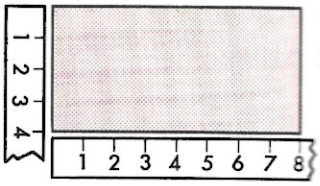
[A] 

[B] 

[C] 

[D] 

[E] 

[F] 

2. Make a table like the one on the right and give the missing numbers. The table refers to exercise 1.

Figure	l (length)	w (width)	A (area)
[A]	6	5	
[B]	8		
[C]		3	
[D]			
[E]			
[F]			

3. Give the areas of rectangles with these lengths and widths.

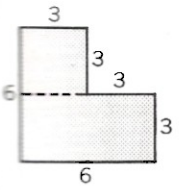
[A] $l = 8$ $w = 7$ [B] $l = 15$ $w = 4$ [C] $l = 16$ $w = 12$

[D] $l = 27$ $w = 38$ [E] $l = 156$ $w = 23$

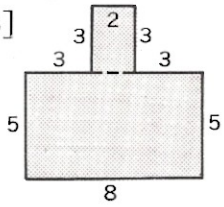
4. Explain this formula: $A = l \times w$

5. Find the area of each figure. Use the lengths given.

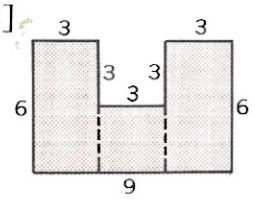
[A]



[B]

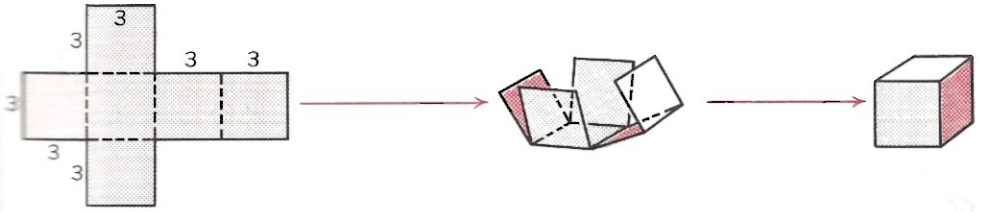


[C]

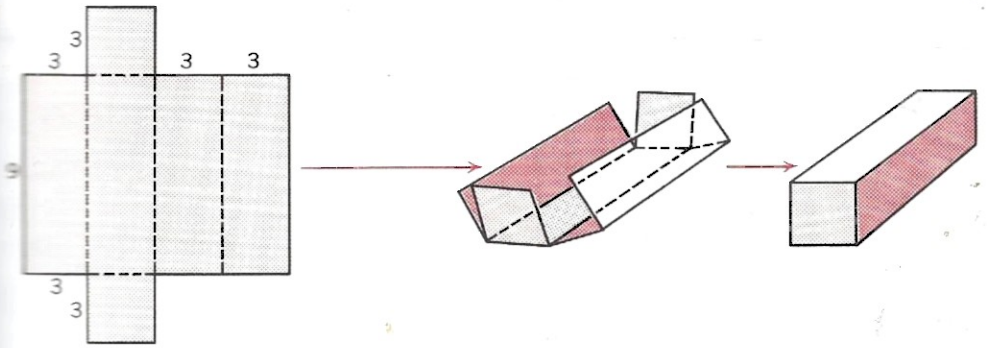


6. Give the area of the plane figure. Then give the surface area of the space figure.

[A]

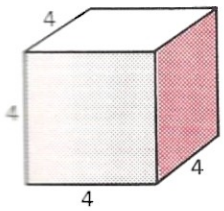


[B]

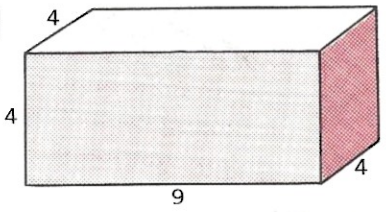


7. Give the surface area of each figure. Use the lengths given.

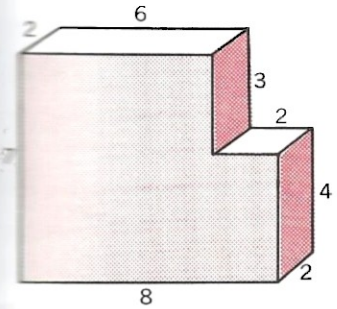
[A]



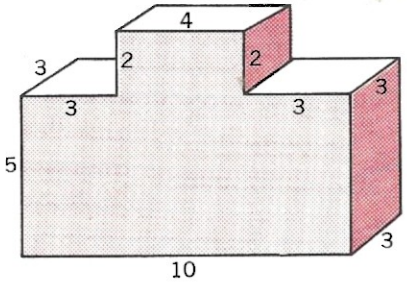
[B]



★[C]



★[D]



CHAPTER 3 | Computing

Finding sums

DISCUSSION EXERCISES

In examples A and B, some of the digits are covered by screens.

A
$$\begin{array}{r} \square 6 \square \\ +\square 7 \square \\ \hline \square\square \end{array}$$

B
$$\begin{array}{r} \square 9 \square\square \\ \square 8 \square\square \\ +\square 5 \square\square \\ \hline \square\square\square \end{array}$$

1. Explain why the number of tens for the sum in A is either 3 or 4.
2. Why can the number of hundreds be 2, 3, or 4 in example B?

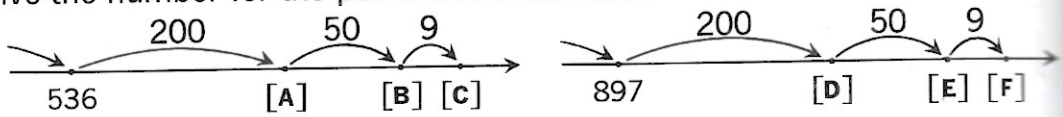
EXERCISES

1. In each example, give the digits that are possible for the red screen.

[A] $\begin{array}{r} 6\square \\ +8\square \\ \hline \square\square \end{array}$ [B] $\begin{array}{r} 4\square\square \\ +3\square\square \\ \hline \square\square \end{array}$ [C] $\begin{array}{r} \square 9 \square \\ \square 8 \square \\ +\square 2 \square \\ \hline \square\square \end{array}$ [D] $\begin{array}{r} 8\square\square \\ \square\square \\ +2\square\square \\ \hline \square\square\square \end{array}$

★[E] $5\square + 7\square = \square\square 9$ ★[F] $6\square\square + \square\square + \square 9\square\square = \square\square\square$

2. Give the number for the point above each letter.



3. Estimate the sum for exercises C, I, K, and N. Then find all sums.

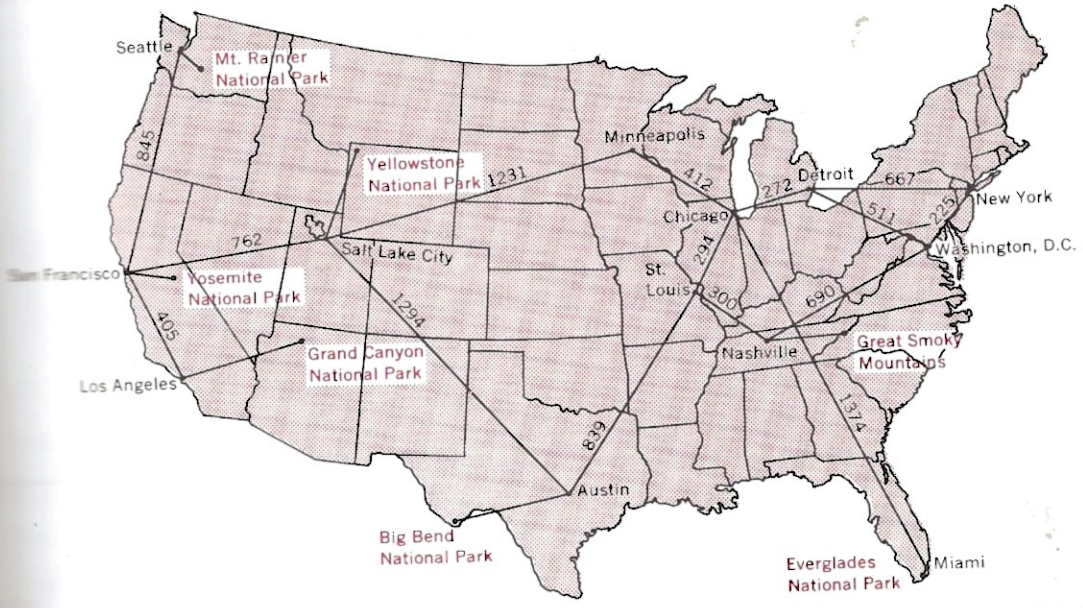
[A] $\begin{array}{r} 28 \\ +57 \\ \hline \end{array}$ [B] $\begin{array}{r} 65 \\ +93 \\ \hline \end{array}$ [C] $\begin{array}{r} 78 \\ +69 \\ \hline \end{array}$ [D] $\begin{array}{r} 96 \\ +54 \\ \hline \end{array}$ [E] $\begin{array}{r} 76 \\ +89 \\ \hline \end{array}$ [F] $\begin{array}{r} 94 \\ +29 \\ \hline \end{array}$

[G] $\begin{array}{r} 64 \\ 23 \\ +92 \\ \hline \end{array}$ [H] $\begin{array}{r} 56 \\ 75 \\ +81 \\ \hline \end{array}$ [I] $\begin{array}{r} 934 \\ 369 \\ +548 \\ \hline \end{array}$ [J] $\begin{array}{r} 675 \\ 87 \\ +948 \\ \hline \end{array}$ [K] $\begin{array}{r} 6328 \\ +5473 \\ \hline \end{array}$ [L] $\begin{array}{r} 3629 \\ +840 \\ \hline \end{array}$

[M] $562 + 78 + 843 + 7651$ [N] $7659 + 8321 + 78 + 642$

[O] $8472 + 9 + 634 + 28$ [P] $7965 + 9374 + 8652 + 85$

Vacation travel



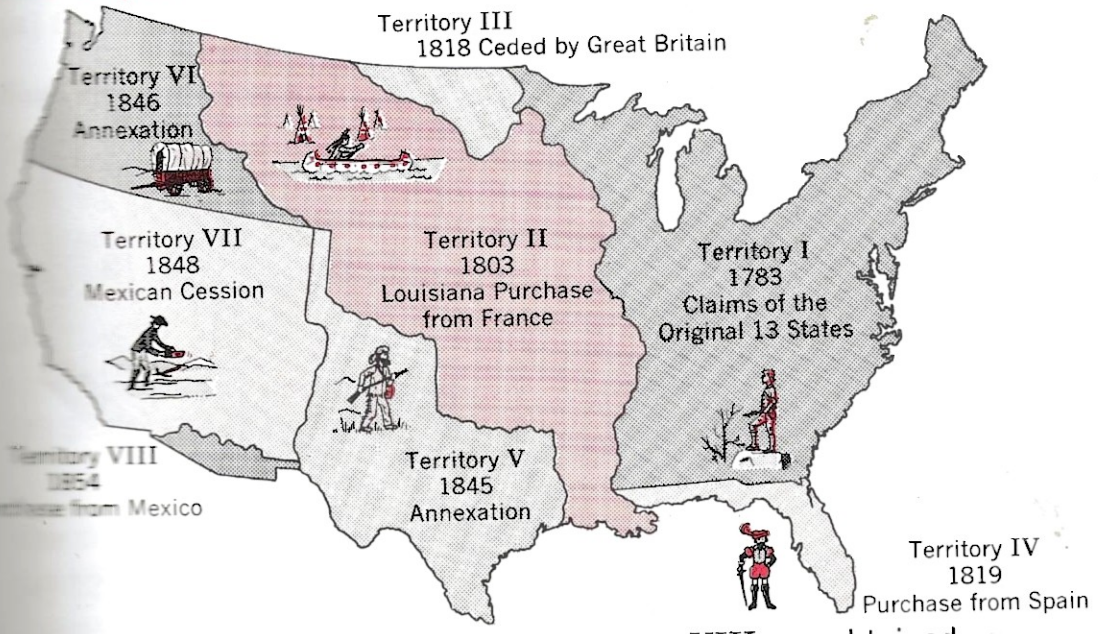
1. Use the map and chart to give the one-way mileage for each trip.
   - [A] Los Angeles to Seattle to Mt. Rainier National Park
   - [B] Minneapolis to San Francisco to Yosemite
   - [C] Seattle to Los Angeles to Grand Canyon
   - [D] Chicago to Austin to Big Bend
   - [E] New York to Nashville to the Smoky Mountains
   - [F] Chicago to the Everglades
   - [G] Austin to Salt Lake City to Yellowstone

Trip	Mileage
Big Bend to Austin	471
Everglades to Miami	75
Grand Canyon to Los Angeles	512
Smoky Mts. to Nashville	290
Mt. Rainier to Seattle	100
Yellowstone to Salt Lake City	317
Yosemite to San Francisco	150

2. Use the map to give the shortest mileages for these trips.
   - [A] Chicago to Yosemite
   - [B] Detroit to the Everglades
   - [C] New York to Yosemite
   - [D] Washington, D.C., to Yellowstone

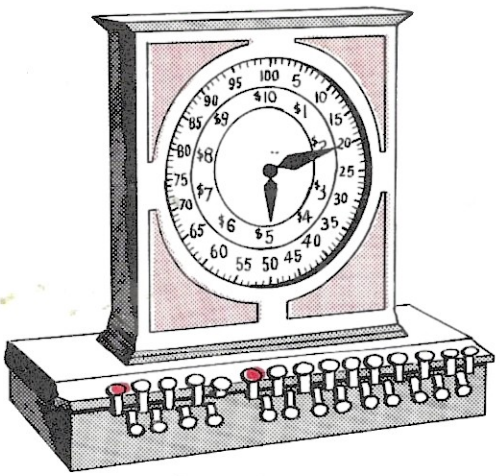
3. Plan a trip from your city to one of the above points of interest. Give the total mileage for this trip. Use only the routes shown and the distance from your city to one of the points on the map.

Land expansion of the United States



1. Alaska was purchased 13 years after Territory VIII was obtained. When was Alaska purchased?
2. Hawaii was obtained in 1898. How long after the purchase of Alaska was this?
3. How long after 1783 was it before the United States obtained more land?
4. Hawaii became a state in 1959. How many years was this from the time it was obtained in 1898?
5. Alaska became a state 92 years after it was purchased. What year was this?
6. The Declaration of Independence was adopted 7 years before claims were made for Territory I. When was the Declaration of Independence adopted?
7. The Civil War started in 1861. How long was this after the adoption of the Declaration of Independence?
8. James Monroe was president when Territory IV was purchased from Spain. He was born 61 years before the purchase of Territory IV and lived to the age of 73. Give the year of birth and the year of death of this famous president.
9. How long after 1783 was it before your state officially became one of the United States?

Money



One of the first cash registers resembled a clock. One hand indicated the number of cents and the other hand indicated the number of dollars. How much is indicated by this register?

Modern cash registers show the amount for each purchase, the total amount, and how much change the buyer should receive. They also keep a record of total purchases, and print tapes showing all amounts and the total.

EXERCISES

1. [A] Suppose a cash register rings up these two purchases. What total should it give?

[B] If you pay with a 20-dollar bill, what should the machine show for your change?

2. If this is the amount of change you get from a 10-dollar bill, what was the amount of your purchase?

3. Suppose this is the total shown for the purchase of 3 items. Give one set of possible prices for these 3 items.

4. [A] Estimate the totals for exercises 5 **A**, **B**, **C**, and **D**.
[B] Estimate the differences for exercises 6 **A**, **B**, **G**, and **H**.

5. Find the total amounts.

[A]	[B]	[C]	[D]	[E]
$2.98	$9.82	$5.68	$15.67	$117.88
3.65	6.54	6.47	83.47	654.92
7.82	7.29	9.98	16.59	846.98

6. Find the differences in the amounts.

[A]	[B]	[C]	[D]	[E]
$5.67	$4.82	$19.21	$765.42	$9.37
2.91	3.46	12.59	186.50	6.54

[F]	[G]	[H]	[I]	[J]
$8.00	$20.00	$100.00	$10.00	$5.00
1.29	16.98	37.98	.75	.98

7. Use dollar notation to write each amount.
[A] 125¢ [B] 700¢ [C] 1298¢ [D] 95¢ [E] 5¢ [F] 10¢ [G] 50¢ [H] 25¢

Short stories—money

1. Had 1 dollar 25 cents.
Spent 79 cents. How much left?

2. Had $12.53.
Earned 5 dollars and
a quarter. How much now?

3. Had 10 dollars. Spent 3 dollars.
Then spent $2.79. How much left?

4. Broke. Went to work and
earned $1.25.
Spent 98 cents. How much now?

5. 15 dollars in the bank.
Withdrew $10.98.
How much left in the bank?

6. $27.35 in the bank.
Deposited $3.95.
How much in the bank now?

7. Had a quarter.
Earned 50 cents.
Spent 35 cents.
How much left?

8. Had a dollar.
Spent 49 cents.
Lost a quarter.
How much left?

9. Broke. Found 5 dollars.
Spent $2.98.
Earned $1.25.
How much now?

10. Had 20 dollars.
Bought 2 things. $1.98 each.
How much left?

Finding products

When one or more of the factors has two digits, you can use the distributive principle to find the product.

$4 \times 76 = (4 \times 70) + (4 \times 6)$		
Step 1 $\begin{array}{r} 76 \\ \times\ 4 \\ \hline 24 \end{array}$	Step 2 $\begin{array}{r} 76 \\ \times\ 4 \\ \hline 24 \\ 280 \end{array}$	Step 3 $\begin{array}{r} 76 \\ \times\ 4 \\ \hline 24 \\ 280 \\ \hline 304 \end{array}$
$4 \times 6 = 24$	$4 \times 70 = 280$	$24 + 280 = 304$

Explain the steps in this short cut.

$$\begin{array}{r} {}^{2}76 \\ \times\ 4 \\ \hline 4 \end{array} \longrightarrow \begin{array}{r} {}^{2}76 \\ \times\ 4 \\ \hline 304 \end{array}$$

$3 \times 487 = (3 \times 400) + (3 \times 80) + (3 \times 7)$			
Step 1 $\begin{array}{r} 487 \\ \times\ 3 \\ \hline 21 \end{array}$	Step 2 $\begin{array}{r} 487 \\ \times\ 3 \\ \hline 21 \\ 240 \end{array}$	Step 3 $\begin{array}{r} 487 \\ \times\ 3 \\ \hline 21 \\ 240 \\ 1200 \end{array}$	Step 4 $\begin{array}{r} 487 \\ \times\ 3 \\ \hline 21 \\ 240 \\ 1200 \\ \hline 1461 \end{array}$
$3 \times 7 = 21$	$3 \times 80 = 240$	$3 \times 400 = 1200$	Add

Explain each step in this short cut.

$$\begin{array}{r} {}^{2}4{}^{2}87 \\ \times\ 3 \\ \hline 1 \end{array} \longrightarrow \begin{array}{r} {}^{2}4{}^{2}87 \\ \times\ 3 \\ \hline 61 \end{array} \longrightarrow \begin{array}{r} {}^{2}4{}^{2}87 \\ \times\ 3 \\ \hline 1461 \end{array}$$

EXERCISES

1. Give the correct sign ($<$ or $>$) for each ◯.

[A] 487×3 ◯ 500×3

[B] 692×4 ◯ 2800

[C] 4000 ◯ 5×769

[D] 612×3 ◯ 3×600

[E] 309×7 ◯ 2100

[F] 2000 ◯ 5×399

[G] 7×899 ◯ 6300

[H] 8000 ◯ 999×8

[I] 2400 ◯ 6×408

2. Estimate the products for exercises 3 B, C, F, H, K, M, P, T, and V.

3. Find the products.

[A] 56 ×3

[B] 72 ×6

[C] 59 ×5

[D] 62 ×8

[E] 47 ×6

[F] 159 ×9

[G] 328 ×6

[H] 527 ×2

[I] 806 ×7

[J] 564 ×4

[K] 392 ×7

[L] 564 ×8

[M] 306 ×7

[N] 530 ×9

[O] 654 ×5

[P] 1326 ×3

[Q] 5874 ×6

[R] 6521 ×5

[S] 7643 ×8

[T] 1087 ×4

[U] 6503 ×9

[V] 8007 ×7

[W] 6493 ×6

4. Find the product for **a**. Then give the product for **b**.

[A] $6 \times 84 = a \rightarrow 60 \times 84 = b$

[B] $7 \times 53 = a \rightarrow 70 \times 53 = b$

[C] $9 \times 61 = a \rightarrow 90 \times 61 = b$

[D] $5 \times 82 = a \rightarrow 50 \times 82 = b$

[E] $6 \times 364 = a \rightarrow 60 \times 364 = b$

[F] $7 \times 156 = a \rightarrow 70 \times 156 = b$

[G] $9 \times 348 = a \rightarrow 90 \times 348 = b$

[H] $8 \times 716 = a \rightarrow 80 \times 716 = b$

5. Find the products.

[A] 48 ×70

[B] 63 ×80

[E] 57 ×30

[D] 69 ×50

[H] 846 ×90

[F] 562 ×30

[M] 675 ×60

[H] 659 ×80

Think Think Think Think

In the equation below, some of the digits are covered by a ▮. There is enough shown to find the two possible numbers for **a**. Find these numbers.

$16▮ \times a = ▮▮1$

Two- and three-digit factors

These examples illustrate again how the distributive principle is used to help you find products.

$38 \times 47 = (30 \times 47) + (8 \times 47)$		
Step 1 47 ×38 376	Step 2 47 ×38 376 1410	Step 3 47 ×38 376 1410 1786
$8 \times 47 = 376$	$30 \times 47 = 1410$	$376 + 1410 = 1786$

$43 \times 627 = (40 \times 627) + (3 \times 627)$		
Step 1 627 ×43 1881	Step 2 627 ×43 1881 25080	Step 3 627 ×43 1881 25080 26961
$3 \times 627 = 1881$	$40 \times 627 = 25{,}080$	$1881 + 25{,}080 = 26{,}961$

EXERCISES

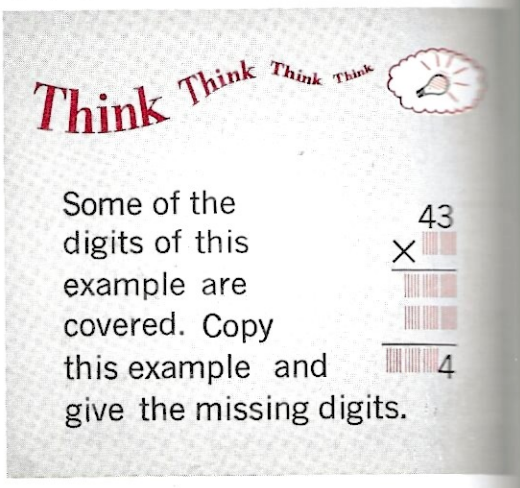
1. Estimate the products for exercises 2 A, E, I, K, and L.

2. Find the products.

[A] 36 ×48 [B] 59 ×27 [C] 78 ×35 [D] 64 ×37

[E] 26 ×39 [F] 342 ×87 [G] 647 ×98 [H] 765 ×77

[I] 826 ×48 [J] 392 ×64 [K] 822 ×41 [L] 607 ×38

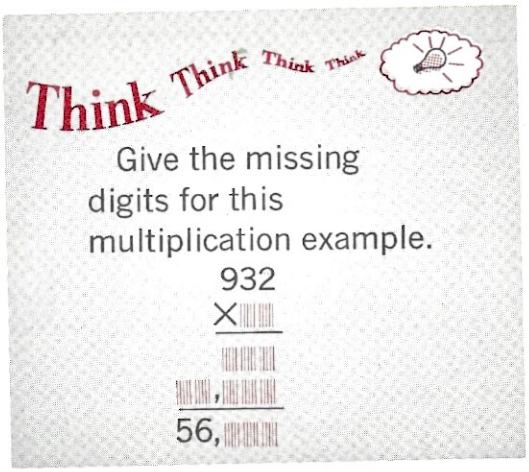


3. For each exercise, give the products for **a** and **b**.

[A] $6 \times 827 = 4962 \longrightarrow \begin{matrix} 60 \times 827 = \mathbf{a} \\ 600 \times 827 = \mathbf{b} \end{matrix}$

[B] $4 \times 391 = 1564 \longrightarrow \begin{matrix} 40 \times 391 = \mathbf{a} \\ 400 \times 391 = \mathbf{b} \end{matrix}$

[C] $7 \times 285 = 1995 \longrightarrow \begin{matrix} 70 \times 285 = \mathbf{a} \\ 700 \times 285 = \mathbf{b} \end{matrix}$



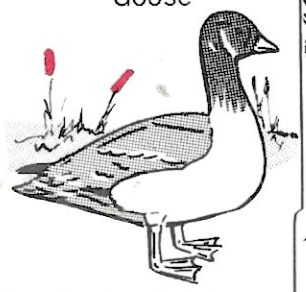
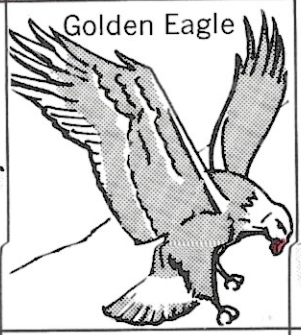
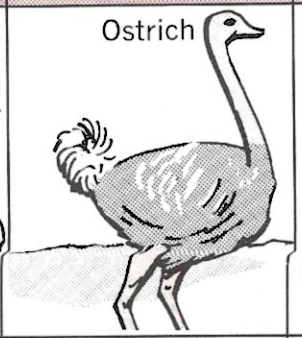
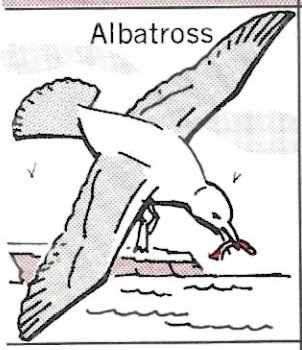
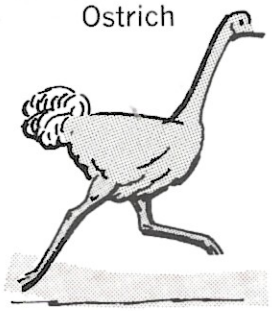
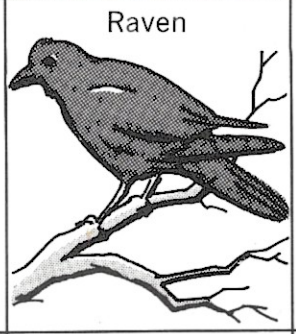
4. Find the products.

[A] 347 ×2	347 ×60	347 ×300	→	347 ×362
[B] 641 ×6	641 ×20	641 ×400	→	641 ×426
[C] 917 ×4	917 ×70	917 ×500	→	917 ×574
[D] 186 ×7		186 ×800	→	186 ×807
[E] 342 ×5	342 ×30	342 ×900	→	342 ×935
[F] 253 ×8		253 ×300	→	253 ×308
[G] 576 ×3	576 ×90	576 ×200	→	576 ×293

5. Find the products.

[A] 384 ×264	[B] 462 ×397	[C] 921 ×846	[D] 847 ×123	[E] 625 ×804	[F] 306 ×357
[G] 570 ×642	[H] 809 ×817	[I] 654 ×156	[J] 928 ×407	[K] 3624 ×846	[L] 7516 ×347
[M] 3207 ×782	[N] 9843 ×260	[O] 6125 ×256	[P] 5280 ×843	[Q] 6407 ×604	[R] 8023 ×834

Birds*

Highest flier	Fastest flier	Largest	Smallest
Goose 	Golden Eagle 	Ostrich 	Hummingbird
26,000 feet	175 mph	300 pounds	2 inches
Greatest wingspread	**Fastest runner**	**Oldest**	**Greatest traveler**
Albatross 	Ostrich 	Raven 	Tern
11 to 12 feet	50 mph	69 years	11,000 miles

1. Many modern jet aircraft fly as high as 35,000 feet. How much higher is this than the highest-flying goose?

2. Some jet aircraft cruise at a speed that is about 225 miles per hour faster than twice as fast as the eagle. How fast is this?

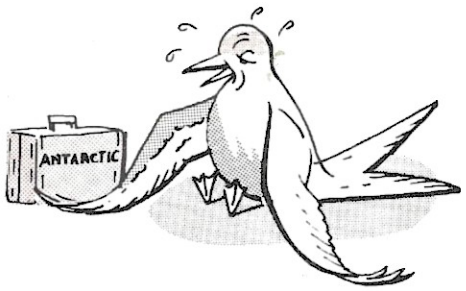
3. How many 2-inch-long hummingbirds would have to be laid end to end to reach across the 12-foot wingspread of the albatross?

4. One of the oldest men on record lived to be 113 years old. How much older is this than the "old" raven?

5. How many ounces does the ostrich weigh?

*All records given are approximations.

6. The arctic tern migrates from the Arctic to the Antarctic (11,000 miles each way) once a year. How far would the tern travel in 6 years?



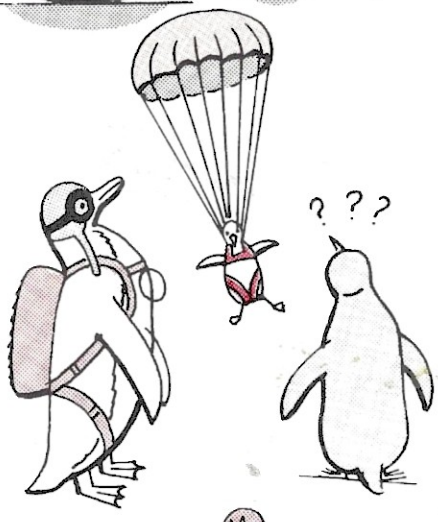
7. There are 1760 yards in a mile. If an ostrich could run at his top speed for an hour, how many yards would he travel?

8. The largest flying bird weighs about 40 pounds. It would take about 192 hummingbirds to weigh 1 pound. How many hummingbirds would it take to weigh 40 pounds?

9. The canvasback duck flies about 20 mph faster than the ostrich can run. How many feet does the duck fly in an hour? (There are 5280 feet in one mile.)

★10. An ostrich egg weighs about 3 pounds. A hen's egg weighs about 2 ounces. It would take 200 hummingbird eggs to weigh 1 ounce. How many hummingbird eggs would it take to weigh as much as one ostrich egg?

★11. The penguin is a bird that does not fly. Some penguins can swim as fast as 20 mph. How many feet per second is this?



★12. Sparrow eggs hatch in about 12 days. It takes 5 times that long for penguin eggs. Chicken eggs take 39 days less than penguin eggs. How much time does it take for penguin eggs and chicken eggs to hatch?

Exponents again

All your work with exponents so far has been with powers of ten. For example, $10^3 = 10 \times 10 \times 10$; $10^2 = 10 \times 10$; $10^4 = 10 \times 10 \times 10 \times 10$. In this lesson, you will learn to use exponents to express powers of other numbers. The following examples illustrate this.

For these symbols	We think	We write
5^3	$5 \times 5 \times 5$	$5^3 = 125$
3^4	$3 \times 3 \times 3 \times 3$	$3^4 = 81$
4^2	4×4	$4^2 = 16$
2^5	$2 \times 2 \times 2 \times 2 \times 2$	$2^5 = 32$
7^3	$7 \times 7 \times 7$	$7^3 = 343$
6^4	$6 \times 6 \times 6 \times 6$	$6^4 = 1296$

This example shows how to use exponents.

$$\begin{cases} a^1 = a \\ a^2 = a \times a \\ a^3 = a \times a \times a \\ a^4 = a \times a \times a \times a \\ \cdot \\ \cdot \\ \cdot \\ a^n = \overbrace{a \times a \times a \times \cdots}^{n \text{ factors of } a} \end{cases}$$

EXERCISES

1. Give the missing numbers.

[A] $6^2 =$ ▒▒ [B] $7^2 =$ ▒▒ [D] $4^3 =$ ▒▒ [F] $3^4 =$ ▒▒ [H] $5^2 =$ ▒▒ [J] $7^1 =$ ▒

(Answer: 36) [C] $2^3 =$ ▒▒ [E] $3^2 =$ ▒▒ [G] $2^4 =$ ▒▒ [I] $6^3 =$ ▒▒ [K] $4^4 =$ ▒

2. Give the correct number for each sentence.

[A] If $a = 7$, then $a^2 =$ ▒▒.

[B] If $x = 3$, then $x^3 =$ ▒▒.

[C] If $m = 4$, then $m^2 =$ ▒▒.

[D] If $r = 2$, then $r^4 =$ ▒▒.

[E] If $t = 5$, then $t^3 =$ ▒▒.

[F] If $b = 6$, then $b^3 =$ ▒▒.

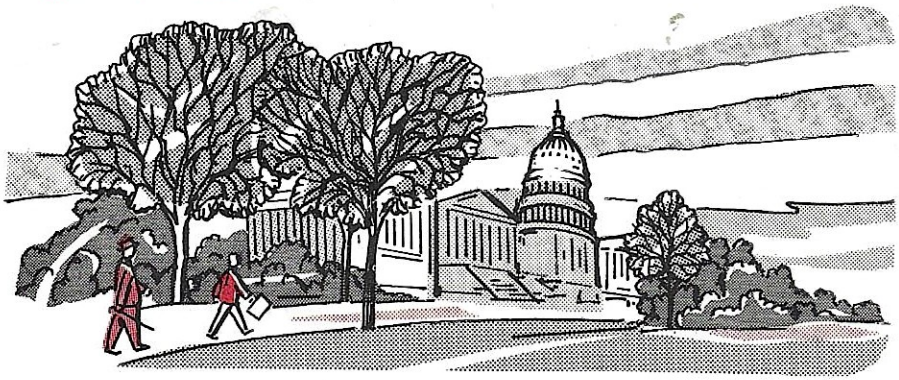
[G] If $c = 3$, then $2 \times c^2 =$ ▒▒.

[H] If $y = 4$, then $4 \times y^3 =$ ▒▒.

[I] If $n = 5$, then $3 \times n^2 =$ ▒▒.

[J] If $d = 7$, then $5 \times d^2 =$ ▒▒.

Washington, D.C.



1. In a recent year the population of Washington, D.C., was 763,956. The population of Washington together with the population of the surrounding suburbs was 2,001,897. What was the population of the surrounding suburbs?

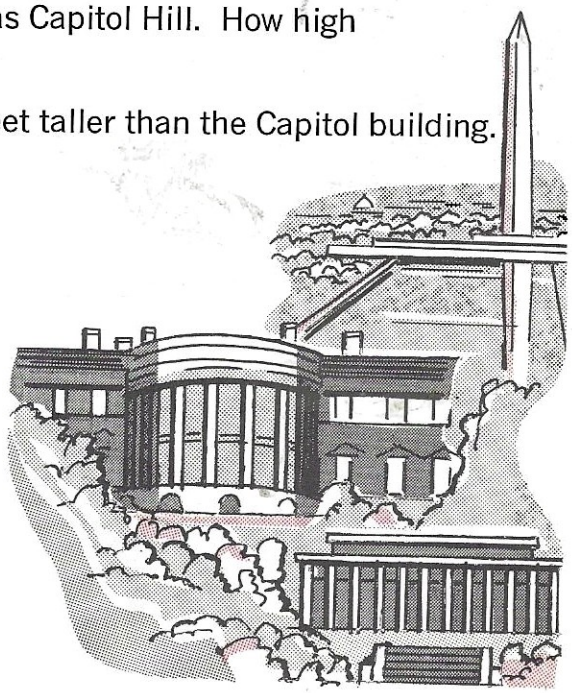
2. Washington covers an area of 69 square miles. Maryland has an area that is about 153 times the area of Washington. About what is the area of Maryland?

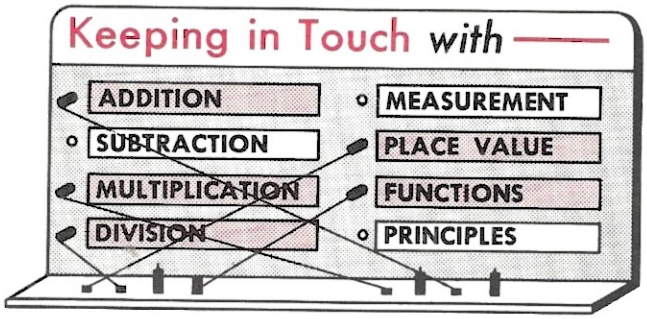
3. The Capitol building stands on Capitol Hill. The hill is only 83 feet high, which seems very low when compared with a structure like the Washington Monument. The monument is about 57 feet higher than 6 times as high as Capitol Hill. How high is the Washington Monument?

4. Washington Monument is just 268 feet taller than the Capitol building. How tall is the Capitol building?

5. The White House is the residence of the President. The main building is 3 stories tall, and its base is a rectangular region that measures 170 feet by 85 feet. What is the area of this region?

6. The Lincoln Memorial is a large marble hall enclosing 3 memorials to President Lincoln. The main memorial is 79 feet 10 inches tall. How many inches is this?





1. Find the quotients.

[A] $24 \div 4$
[B] $240 \div 4$
[C] $54 \div 9$
[D] $540 \div 9$
[E] $5400 \div 9$
[F] $63 \div 7$
[G] $6300 \div 7$
[H] $42 \div 6$
[I] $420 \div 6$
[J] $4200 \div 6$

2. Find the largest possible number for **n**.

[A] $n \times 40 < 243$
[B] $n \times 50 < 360$
[C] $n \times 80 < 331$
[D] $n \times 20 < 161$
[E] $n \times 30 < 220$
[F] $n \times 60 < 375$
[G] $n \times 40 < 375$
[H] $n \times 90 < 730$
[I] $n \times 70 < 400$
[J] $n \times 30 < 100$
[K] $n \times 40 < 300$
[L] $n \times 60 < 500$

3. Find the largest multiple of 10 for **n**.

[A] $n \times 20 < 1150$
[B] $n \times 60 < 2300$
[C] $n \times 50 < 2400$
[D] $n \times 70 < 6100$
[E] $n \times 30 < 1280$
[F] $n \times 40 < 3300$
[G] $n \times 50 < 2700$
[H] $n \times 30 < 2500$
[I] $n \times 80 < 3700$
[J] $n \times 90 < 5800$
[K] $n \times 70 < 6100$
[L] $n \times 60 < 5700$

4. Give the number for **n**.

[A] $3682 = (n \times 100) + 82$
[B] $5823 = (n \times 10) + 3$
[C] $8264 = (n \times 10) + 4$
[D] $9176 = (n \times 100) + 76$
[E] $84{,}327 = (n \times 100) + 27$
[F] $93{,}982 = (n \times 100) + 82$
[G] $74{,}656 = (n \times 100) + 56$
[H] $60{,}432 = (n \times 10) + 2$

5. For each of the following equations, tell whether the solution is a 1-, 2-, or 3-digit number.

[A] $n \times 35 = 805$
[B] $n \times 35 = 8050$
[C] $n \times 47 = 94$
[D] $n \times 47 = 940$
[E] $n \times 47 = 9400$
[F] $n \times 26 = 182$
[G] $n \times 26 = 1820$
[H] $n \times 76 = 684$
[I] $n \times 76 = 6840$
[J] $n \times 97 = 582$
[K] $n \times 34 = 2720$
[L] $n \times 83 = 58{,}100$

6. Complete the table.

function rule $(8 \times n) + 9$

	n	$f(n)$
[A]	6	
[B]	0	
[C]	9	
[D]		25
[E]	7	
[F]		49

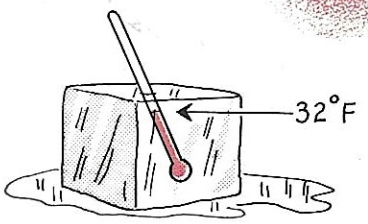
63 ÷ 7
6300 ÷ 7
42 ÷ 6
420 ÷ 6
4200 ÷ 6

400
100
300
500

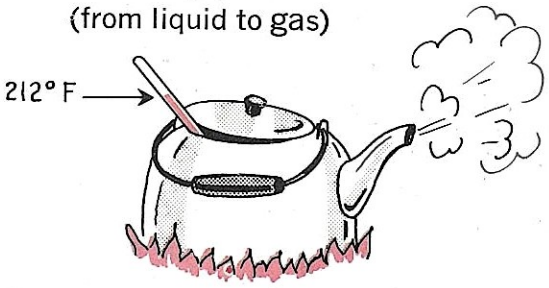
3700
5800
6100
5700

Temperature

Melting point*
(from solid to liquid)



Boiling point*
(from liquid to gas)



1. Iron melts at 2802°. How much hotter is this than the melting point of ice?

2. Lead melts at a temperature that is 1181° cooler than that for iron. What is this melting point?

3. The melting point of gold is about 61 times the melting point of ice. Give this temperature.

4. The melting point of paraffin is 108° greater than the melting point of ice. Give this temperature.

5. Tungsten can be used in light bulbs since it has a very high melting point. This melting point is about 193 times the melting point of ice. Find it.

★6. Mercury is liquid at room temperature. It has a melting point of 39° **below** zero. How much does this differ from the melting point of ice?

7. The boiling point of alcohol is 172°. How much less is this than the boiling point of water?

8. Metals have a boiling point just as they have a melting point. Iron boils at a temperature that is 2158° hotter than its melting point. What is the boiling point of iron?

9. Gold boils at a temperature that is 1490° hotter than twice its melting point. (See exercise 3.) Give the boiling point of gold.

10. Changes in pressure change the boiling points of liquids. On a 10,000-foot mountain, water boils at a temperature that is 18° cooler than the boiling point given above. Give this temperature.

★11. Mercury boils at 675°. What is the difference between the melting and boiling points of mercury?

*At sea level, in Fahrenheit degrees.

Dividing

We find the **quotient** by subtracting multiples of the **divisor** from the **dividend.**

Quotient → 726
Divisor → 6)4356
Dividend →

DISCUSSION EXERCISES

1. This example shows that to find $4356 \div 6$, we find the greatest number of sixes that can be subtracted from 4356.

[A] How many sixes were subtracted the first time?

[B] How many sixes were subtracted in all?

[C] What is the quotient?

[D] What is the remainder?

```
  726
6)4356
 -4200   ← 700 × 6
   156
  -120   ←  20 × 6
    36
   -36   ←   6 × 6
     0
```

2. This example shows two solutions to the same problem. Some of the numerals have been covered by screens. The numbers written in the circles are a record of the number of eights subtracted each time. Copy each solution and give the missing numbers. Which solution do you think is easier? Explain why.

Solution 1

```
8) 1883
 -800   (100)
 1083
 -800   (   )
  283
 -160   (   )
  123
  -80   (10)
   43
  -40   (   )
    3
```

Solution 2

```
8) 1883
-1600   (   )
  283
 -240   (   )
   43
  -40   (   )
    3
```

3. Copy each problem. Complete the solution to find the quotient. Explain your solution.

[A] $9\overline{)5463}$
-5400 (600)
63

[B] $40\overline{)387}$
-200 (5)
187

[C] $39\overline{)261}$
-234 ( )

[D] $53\overline{)1325}$
-1060 (20)

Short cuts–divisors less than 10

In your work in dividing, you should look for the most efficient way to find the quotient. The method shown in the examples (often called "short division") is an easy way to find the quotient when the divisor is less than 10. Compare each step in example A with exercise 1, page 60.

A

Step 1

Thinking about **Hundreds**: $43 \div 6$
Quotient, 7
Remainder, 1

$$6\overline{)4\,3^{1}5\,6}\quad 7$$

Step 2

Thinking about **tens**: $15 \div 6$
Quotient, 2
Remainder, 3

$$6\overline{)4\,3^{1}5^{3}6}\quad 7\,2$$

Step 3

Thinking about **ones**: $36 \div 6$
Quotient, 6
Remainder, 0

$$6\overline{)4\,3^{1}5^{3}6}\quad 7\,2\,6$$

Check

$$\begin{array}{r} 726 \\ \times 6 \\ \hline 4356 \end{array}$$

B

Step 1

Thinking about **Hundreds**:
$16 \div 8 = 2$

$$8\overline{)1\,6\,4\,9}\quad 2$$

Step 2

Thinking about **tens**: There are 0 eights in 4.
The remainder is 4.

$$8\overline{)1\,6\,4\,9}\quad 2\,0$$

Step 3

Thinking about **ones**: $49 \div 8$
Quotient, 6
Remainder, 1

$$8\overline{)1\,6\,4\,9}\quad 2\,0\,6\ \text{R1}$$

Check

$$\begin{array}{r} 206 \\ \times 8 \\ \hline 1648 \end{array} \qquad \begin{array}{r} 1648 \\ +1 \\ \hline 1649 \end{array}$$

EXERCISES

Find the quotients and remainders.

1. $2\overline{)568}$ 2. $5\overline{)755}$ 3. $4\overline{)2964}$ 4. $3\overline{)6389}$ 5. $7\overline{)6321}$

7. $9\overline{)6584}$ 8. $6\overline{)4224}$ 9. $4\overline{)3478}$ 10. $8\overline{)6666}$

12. $5\overline{)5305}$ 13. $6\overline{)39,872}$ 14. $9\overline{)65,363}$ 15. $3\overline{)18,654}$

Divisors less than 100

In example A, the quotient $213 \div 29$ was found by guessing multiples of 29 to subtract from 213. In example B, a carefully chosen estimate was helpful in finding the quotient. (The sign $\leq$ means "is less than or equal to.")

A

$$\begin{array}{r} 7 \\ 29\overline{)\,213} \\ -87 \quad (3) \\ \hline 126 \\ -87 \quad (3) \\ \hline 39 \\ -29 \quad (1) \\ \hline 10 \end{array}$$

B

$n \times 30 \leq 213$
7 is the largest number for **n**.

$$\begin{array}{r} 7 \\ 29\overline{)\,213} \\ -203 \quad (7) \\ \hline 10 \end{array}$$

Study example B to see how skill in estimation will help you find a quotient more quickly and easily than the method shown in example A. The discussion exercises will help you review some ideas about estimation.

DISCUSSION EXERCISES

1. Explain why 30 was used to help estimate the quotient in example B. Explain example B.

2. **28**$\overline{)126}$ **27**$\overline{)126}$ **26**$\overline{)126}$ **25**$\overline{)126}$

Since **30** is "close" to each of the divisors above, we usually use 30 to help estimate quotients in problems such as these. Explain how 30 is used to estimate the quotients above. Find each quotient and remainder. Explain your work.

3. **31**$\overline{)267}$ **32**$\overline{)267}$ **33**$\overline{)267}$ **34**$\overline{)267}$

Since **30** is "close" to each of the divisors above, we usually use 30 to help estimate quotients in problems such as these. Explain the estimation and your work in finding the quotients and remainders.

4. State a general rule for choosing multiples of 10 to use in estimating quotients.

5. The inequality sentences below will help you estimate the quotient for the three examples. In example A the largest number for **n** is the correct quotient. In example B it is too large, and in example C it is too small. Explain how to complete each example.

(A) $n \times 40 \leq 294$ → $38\overline{)294}$

(B) $n \times 60 \leq 372$ → $64\overline{)372}$

(C) $n \times 30 \leq 208$ → $26\overline{)208}$

EXERCISES

1. Find the quotients and remainders. Check your work.

[A] $40\overline{)234}$	[B] $53\overline{)463}$	[C] $35\overline{)247}$	[D] $15\overline{)124}$	[E] $39\overline{)317}$
[F] $29\overline{)178}$	[G] $77\overline{)658}$	[H] $76\overline{)692}$	[I] $45\overline{)300}$	[J] $17\overline{)153}$
[K] $52\overline{)467}$	[L] $39\overline{)210}$	[M] $44\overline{)246}$	[N] $13\overline{)85}$	[O] $65\overline{)550}$
[P] $38\overline{)293}$	[Q] $61\overline{)537}$	[R] $23\overline{)107}$	[S] $85\overline{)637}$	[T] $14\overline{)97}$
[U] $50\overline{)371}$	[V] $60\overline{)500}$	[W] $17\overline{)102}$	[X] $94\overline{)758}$	[Y] $12\overline{)113}$

2. Find the divisor for each exercise. The quotient is given.

[A] $\blacksquare\overline{)648}$	[B] $\blacksquare\overline{)564}$	[C] $\blacksquare\overline{)294}$	[D] $\blacksquare\overline{)177}$	[E] $\blacksquare\overline{)504}$
72	94	42	59	63

★3. For each exercise, find one number that can serve as both quotient and divisor.

[A] $\blacksquare\overline{)100}$	[B] $\blacksquare\overline{)144}$	[C] $\blacksquare\overline{)400}$
[D] $\blacksquare\overline{)900}$	[E] $\blacksquare\overline{)225}$	[F] $\blacksquare\overline{)625}$
[G] $\blacksquare\overline{)576}$	[H] $\blacksquare\overline{)256}$	[I] $\blacksquare\overline{)1600}$

Think Think Think Think Think

Mercury

Since it takes the planet Mercury 88 of our days to travel once around the sun, a "year" on Mercury is 88 of our days. If a person were 10 years old, give his age in "Mercury years." Use 365 days for an "earth year."

Divisors and quotients less than 100

DISCUSSION EXERCISES

1. Here is the way Jane found the quotient for $512 \div 16$. She thought, "Since $100 \times 16 = 1600$, I know the quotient is less than 100. I will subtract 10 sixteens at a time. Then I will subtract 1 sixteen at a time."

$$\begin{array}{r|l} 16\overline{)\,512} & \\ -160 & (10) \\ \hline 352 & \\ -160 & (10) \\ \hline 192 & \\ -160 & (10) \\ \hline 32 & \\ -16 & (1) \\ \hline 16 & \\ -16 & (1) \\ \hline 0 & \end{array}$$

[A] Explain how Jane found the quotient.

[B] Suppose Jane always uses the method above to find quotients. Explain how she would find the quotient for $1204 \div 14$.

[C] Explain an easier way than Jane's method to find the quotients for $16\overline{)512}$ and $14\overline{)1204}$.

2. If you become skillful in estimating quotients, you will be able to find quotients more easily. Study the example. To find the quotient for $3990 \div 58$, think → $? \times 60 \leq 3990$

Step 1

$$\begin{array}{r|l} 58\overline{)\,3990} & \\ -3480 & (60) \\ \hline 510 & \end{array}$$

[A] What is the largest multiple of 10 that will make the sentence true?

[B] Explain how the number you found in part A is used in step 1 of the example.

[C] To find $510 \div 58$ and complete the dividing, think → $? \times 60 \leq 510$

[D] What is the largest number that will make the sentence true?

Step 2

$$\begin{array}{r|l} & 68 \\ 58\overline{)\,3990} & \\ -3480 & (60) \\ \hline 510 & \\ -464 & (8) \\ \hline 46 & \end{array}$$

[E] Explain how the number you found in part D is used in step 2 to complete the dividing.

[F] What are the quotient and remainder? Check your work.

3. When you estimate quotients, you will find that your estimate does not always give the correct number. Often your estimate will be too large, and you will have to choose a smaller number and try again. Other times your estimate will be too small, and you will have to subtract an extra time. Each of these cases is illustrated in the examples below. *In each example, find the largest multiple* of 10 for **n**. Then use this estimate and explain how to complete the dividing.

A $n \times 50 \leq 1395$

$45\overline{)1395}$

B $n \times 10 \leq 936$

$13\overline{)936}$

C $n \times 30 \leq 900$

$27\overline{)900}$

EXERCISES

1. Find the quotients and remainders. Check your work.

[A] $19\overline{)1670}$ [B] $35\overline{)2586}$ [C] $15\overline{)483}$ [D] $77\overline{)2567}$

[E] $52\overline{)835}$ [F] $17\overline{)554}$ [G] $43\overline{)3000}$ [H] $56\overline{)3875}$

[I] $31\overline{)2863}$ [J] $13\overline{)876}$ [K] $16\overline{)1253}$ [L] $98\overline{)1000}$

[M] $79\overline{)894}$ [N] $12\overline{)158}$ [O] $85\overline{)6550}$ [P] $18\overline{)907}$

[Q] $26\overline{)1372}$ [R] $14\overline{)938}$

2. Find the quotients and remainders.

[A] $5\overline{)4000}$ [B] $30\overline{)500}$

[C] $7\overline{)5836}$ [D] $9\overline{)80{,}000}$

[E] $62\overline{)494}$ [F] $8\overline{)30{,}000}$

[G] $80\overline{)700}$ *[H] $11\overline{)558}$*

[I] $3\overline{)5967}$ [J] $16\overline{)349}$

[K] $6\overline{)38{,}495}$ [L] $15\overline{)750}$

Think Think Think Think

A rock dropped from the top of a tall building will fall about:

16×1^2 ft in 1 sec
16×2^2 ft in 2 sec
16×3^2 ft in 3 sec
$16 \times n^2$ ft in n sec

(Using any number for **n**)

How many seconds (to the nearest second) does it take a rock to fall from the top of the Empire State Building to the ground (1250 feet)?

Average–arithmetic mean

You can think of the average of a set of numbers as a single number that "represents" the numbers in the set. For example, consider the table showing the ages of the first 5 presidents when they took office. Since 58 will substitute for each of the numbers and give the same sum, we call **58** the **average**, or the **arithmetic mean**, of the ages.

President	Age upon entering office
Washington	57
J. Adams	61
Jefferson	57
Madison	57
Monroe	58

58 58 58 58 58

~~57~~ + ~~61~~ + ~~57~~ + ~~57~~ + ~~58~~ = 290

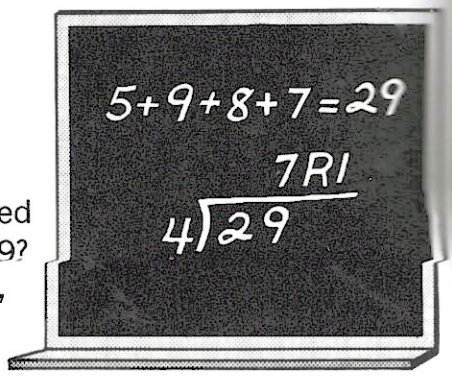
DISCUSSION EXERCISES

1. Explain how you could have found the **arithmetic mean** in the example above.

2. George Washington died at age 67; John Adams, at age 90; and Thomas Jefferson, at 83. Explain how to find the arithmetic mean of these ages.

3. Give the missing words and give an example to explain what the following sentence means. To find the arithmetic mean of a set of numbers, divide the _?_ of the numbers by the _?_ of addends.

4. The chalkboard shows Tom's work in finding the arithmetic mean of 4 numbers.

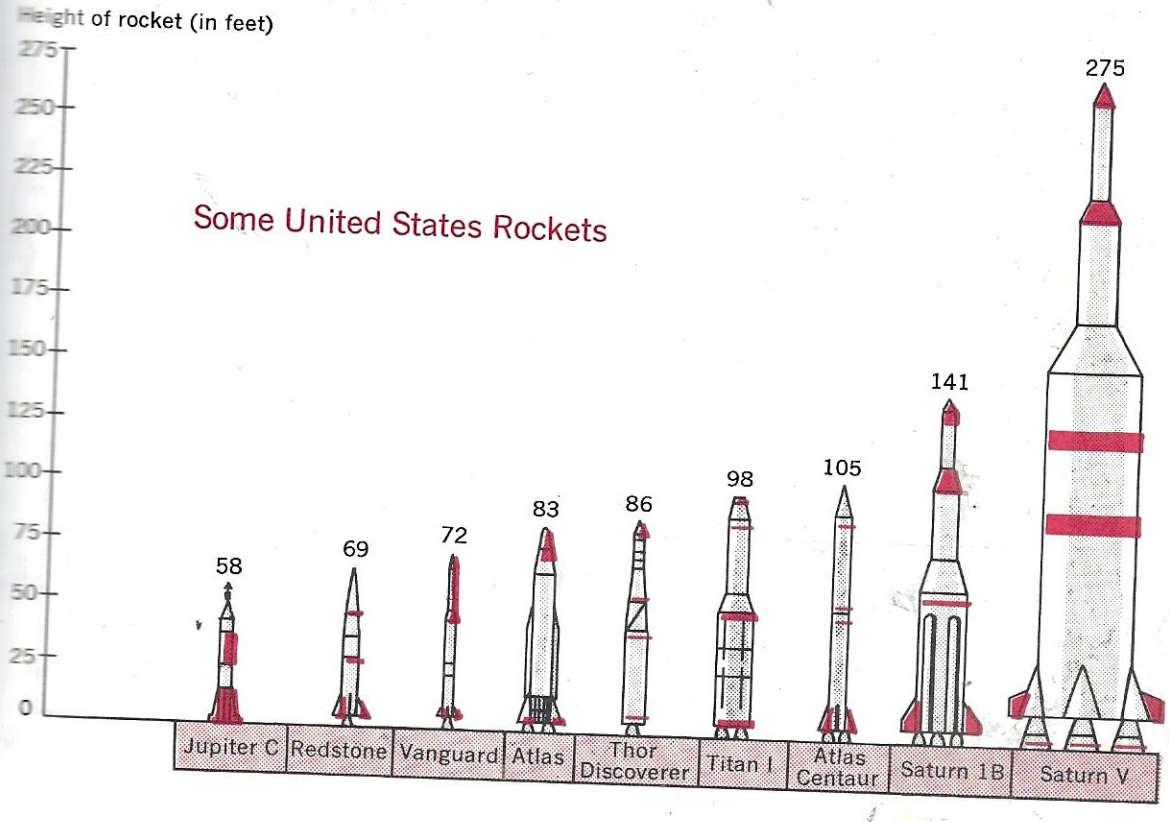


   [A] If you substitute 7 for each addend, will the sum be the same?

   [B] Can any other whole number be substituted for each addend to give a sum closer to 29? We say that the arithmetic mean of 5, 9, 8, and 7 is 7 (to the nearest whole number).

5. Explain how to find the average of 9, 4, 7, 8, and 5 (to the nearest whole number).

Rockets



1. Estimate the average height of the 8 smallest rockets. Then compute the average height (to the nearest whole number).

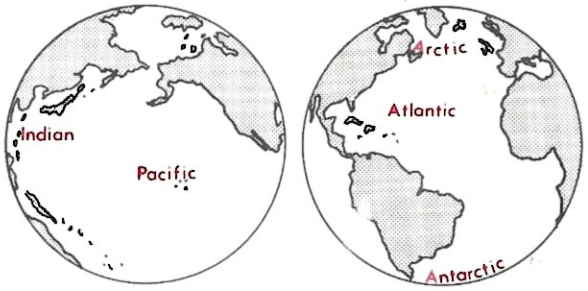
2. Give the arithmetic mean of the heights of all the rockets (to the nearest whole number).

3. About how many Titan I rockets could you lay end to end to make a string of rockets one mile long?

4. It is about 220,000 miles to the moon. If it takes a space ship about 40 hours to make the trip, give the average speed of the ship in miles per hour.

5. One of the Atlas rockets weighed 260,000 pounds at take-off. The Titan and Jupiter rockets weighed 220,000 and 105,000 pounds.
   [A] How many tons did the Atlas rocket weigh?
   [B] Give the average weight of these 3 rockets at take-off.

The oceans



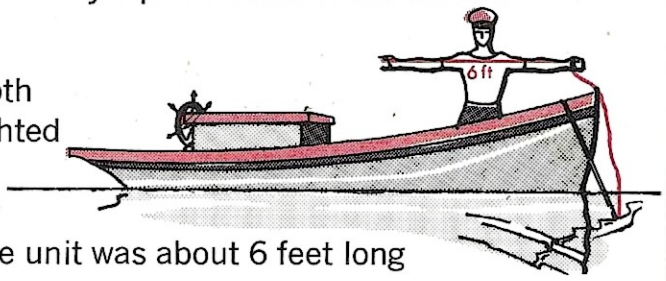
The sea contains about 330 million cubic miles of water. This water contains billions of plants and animals and tons of minerals. The sea is one of the great untapped natural resources for water, food, power, and valuable minerals.

Ocean	Approximate Area (in square miles)
Pacific	68,802,000
Atlantic	31,530,000
Indian	28,356,000
Antarctic	5,731,000
Arctic	5,440,000

1. How many square miles larger is the Antarctic Ocean than the Arctic Ocean?

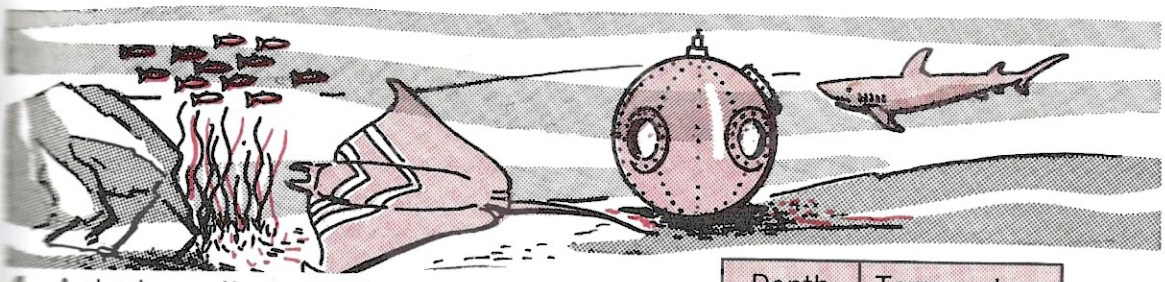
2. The total area of the earth's surface is about 196,950,000 square miles. The total surface area covered by the oceans can be found from the table above. About how many square miles of the earth's surface is covered by land?

3. Sailors used to measure the depth of the ocean by dropping a weighted rope overboard. As they pulled it up, they measured using their outstretched arms as a unit. The unit was about 6 feet long and was called a **fathom.**



[A] The deepest part of the ocean is 35,800 feet. How many fathoms? How many miles (to the nearest mile)?

[B] The sea is 13,560 feet deep at the North Pole. How many fathoms?



4. A device called a bathythermograph is used to record underwater temperatures. The chart shows temperatures recorded every 600 feet in a certain spot in the ocean. At greater depths the temperature usually stays about the same, from 32° to 35°F. Give the average temperature in the table.

Depth (in feet)	Temperature (°F)
600	61°
1200	50°
1800	45°
2400	42°
3000	40°
3600	39°
4200	38°
4800	37°
5400	36°

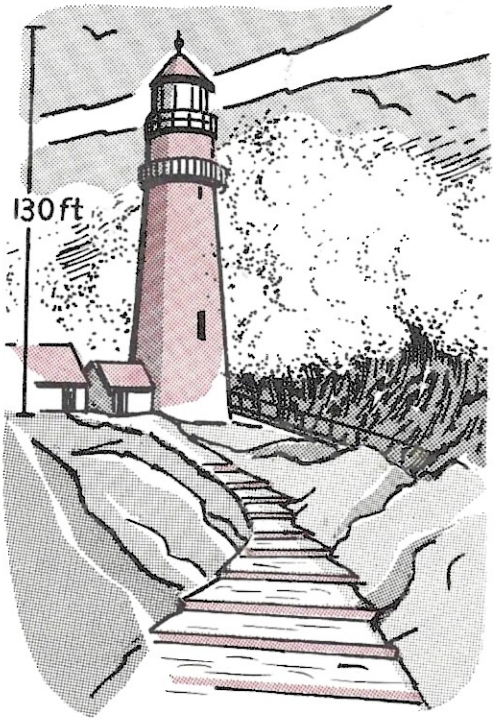
5. A cubic yard of sea water contains about 730 ounces of salt. Salt is often purchased in 26-ounce containers. How many of these containers could be filled with the salt from a cubic yard of sea water?

6. Waves as high as 112 feet have been reported on the "high seas." If each floor of a building is 14 feet tall, the wave would be as tall as a building with how many floors?

7. A cubic mile of sea water contains about 38 pounds of gold.

   [A] How many cubic miles of water would be needed to have 1596 pounds of gold?

   ★[B] One pound of gold will make a thin wire about 900 miles long. The distance around the earth is 24,902 miles. With the gold from 1 cubic mile of sea water, how many feet of wire would you have left after you wrapped it around the earth once?



Short cuts–divisors between 10 and 100

You can find the 3-digit quotient for $39\overline{)22{,}576}$ by finding the 1-digit quotients for:

(A) $39\overline{)225}$ (B) $39\overline{)307}$ (C) $39\overline{)346}$

Find these quotients. Then study the example.

(1) Dividing hundreds

$$\begin{array}{r} 5 \\ 39\overline{)\,22{,}576} \\ -\,195 \\ \hline 30 \end{array}$$

(2) Dividing tens

$$\begin{array}{r} 57 \\ 39\overline{)\,22{,}576} \\ -\,195 \\ \hline 307 \\ -\,273 \\ \hline 34 \end{array}$$

(3) Dividing ones

$$\begin{array}{r} 578 \\ 39\overline{)\,22{,}576} \\ -\,195 \\ \hline 307 \\ -\,273 \\ \hline 346 \\ -\,312 \\ \hline 34 \end{array}$$

For any dividing problem the quotient, no matter how large, can be found by finding the 1-digit quotients for a series of simpler problems.

DISCUSSION EXERCISES

1. Explain how the solutions to A, B, and C above were used to find the quotient in the example.
2. Explain why step 1 in the example above is labeled "dividing hundreds."
3. Explain why step 2 in the example above is labeled "dividing tens."
4. Explain why step 3 in the example above is labeled "dividing ones."
5. Explain why we did not "divide thousands" in the example above.
6. Study these examples. For example A, what division problem is worked when dividing tens? ones? For example B, what division problem is worked when dividing hundreds? tens? ones? Explain each example.

(A)
$$\begin{array}{r} 34 \\ 18\overline{)\,627} \\ -54 \\ \hline 87 \\ -72 \\ \hline 15 \end{array}$$

(B)
$$\begin{array}{r} 416 \\ 72\overline{)\,29{,}952} \\ -288 \\ \hline 115 \\ -72 \\ \hline 432 \\ -432 \end{array}$$

More short cuts

Since all division problems with divisors between 10 and 100 can be solved by finding quotients such as $31\overline{)264}$, it will be helpful to look at a simple way to estimate these 1-digit quotients.

A $12\overline{)49}$

B $\overset{60}{59\overline{)267}}$

C $71\overline{)573}$

D $\overset{20}{18\overline{)173}}$

E $\overset{70}{65\overline{)435}}$

F $34\overline{)197}$

G $\overset{50}{46\overline{)358}}$

H $83\overline{)746}$

DISCUSSION EXERCISES

1. Explain how the numbers indicated by the heavy black numerals can help you estimate the quotients in the examples above.
2. Copy each example above. Use your estimate and find the quotient and remainder.
3. Explain the estimations and the short cut used to find the quotient in example I and in example J.

I Dividing tens

$$\begin{array}{r} \overset{20}{}8 \\ 19\overline{)\,1627} \\ -\,152 \\ \hline 10 \end{array}$$

Dividing ones

$$\begin{array}{r} \overset{20}{}85 \\ 19\overline{)\,1627} \\ -\,152 \\ \hline 107 \\ -\,95 \\ \hline 12 \end{array}$$

J Dividing hundreds

$$\begin{array}{r} 2 \\ 53\overline{)\,13038} \\ -\,106 \\ \hline 24 \end{array}$$

Dividing tens

$$\begin{array}{r} 24 \\ 53\overline{)\,13038} \\ -\,106 \\ \hline 243 \\ -\,212 \\ \hline 31 \end{array}$$

Dividing ones

$$\begin{array}{r} 246 \\ 53\overline{)\,13038} \\ -\,106 \\ \hline 243 \\ -\,212 \\ \hline 318 \\ -\,318 \\ \hline \end{array}$$

EXERCISES

Use the short cut to find the quotients.

1. $61\overline{)26{,}535}$
2. $18\overline{)2232}$
3. $49\overline{)25{,}780}$
4. $93\overline{)22{,}150}$
5. $17\overline{)14{,}235}$
6. $85\overline{)9642}$
7. $34\overline{)21{,}537}$
8. $56\overline{)30{,}764}$

Zeros in the quotient

DISCUSSION EXERCISES

1. [A] Explain how to estimate the number of **hundreds** in the quotient.

[B] Explain the work shown in step 1.

[C] Explain how you can tell that there are 0 **tens** in the quotient.

[D] Explain how to estimate the number of **ones** in the quotient.

[E] Explain step 3.

[F] What is the quotient? What is the remainder?

[G] Explain how to check your work.

2. [A] Explain how to estimate the number of **hundreds** in the quotient.

[B] Explain step 1.

[C] Explain how to estimate the number of **tens** in the quotient.

[D] Explain step 2.

[E] Explain how you can tell that there are 0 **ones** in the quotient.

[F] Give the quotient and remainder. Check the work.

❶ **Step 1** — Dividing hundreds

```
      2
68 ) 1 4 0 9 6
   - 1 3 6
     -----
       4
```

Step 2 — Dividing tens

```
      2 0
68 ) 1 4 0 9 6
   - 1 3 6
     -----
       4 9
```

Step 3 — Dividing ones

```
      2 0 7
68 ) 1 4 0 9 6
   - 1 3 6
     -----
       4 9 6
     - 4 7 6
       -----
         2 0
```

❷ **Step 1** — Dividing hundreds

```
      3
14 ) 4 9 1 1
   - 4 2
     -----
       7
```

Step 2 — Dividing tens

```
      3 5
14 ) 4 9 1 1
   - 4 2
     -----
       7 1
     - 7 0
       -----
         1
```

Step 3 — Dividing ones

```
      3 5 0
14 ) 4 9 1 1
   - 4 2
     -----
       7 1
     - 7 0
       -----
         1 1
```

EXERCISES

Find the quotients and remainders.

1. $68\overline{)20{,}740}$
2. $32\overline{)23{,}685}$
3. $17\overline{)8615}$
4. $45\overline{)13{,}500}$
5. $13\overline{)3521}$
6. $18\overline{)65{,}367}$

Dividing exercises

1. Find the quotients and remainders. Check your work.

[A] $7\overline{)46{,}527}$	[B] $8\overline{)31{,}685}$	[C] $40\overline{)21{,}654}$	[D] $89\overline{)30{,}260}$
[E] $75\overline{)835}$	[F] $18\overline{)3726}$	[G] $31\overline{)8990}$	[H] $56\overline{)3000}$
[I] $64\overline{)52{,}300}$	[J] $5\overline{)80{,}000}$	[K] $19\overline{)3847}$	[L] $13\overline{)9652}$
[M] $55\overline{)35{,}862}$	[N] $95\overline{)38{,}855}$	[O] $17\overline{)5246}$	[P] $47\overline{)31{,}762}$

2. Study the example. Then complete the exercises.

Dividing thousands

$$37\overline{)89355}\quad 2,\ 74,\ 15$$

Dividing hundreds

$$37\overline{)89355}\quad 24,\ 74,\ 153,\ 148,\ 5$$

Dividing tens

$$37\overline{)89355}\quad 241,\ 74,\ 153,\ 148,\ 55,\ 37,\ 18$$

Dividing ones

$$37\overline{)89355}\quad 2415,\ 74,\ 153,\ 148,\ 55,\ 37,\ 185,\ 185$$

[A] $29\overline{)64{,}932}$	[B] $61\overline{)129{,}684}$	[C] $18\overline{)40{,}000}$	[D] $42\overline{)326{,}572}$
[E] $38\overline{)74{,}987}$	[F] $23\overline{)176{,}238}$	[G] $57\overline{)64{,}593}$	[H] $75\overline{)246{,}483}$

3. Study the examples. Then complete the exercises.

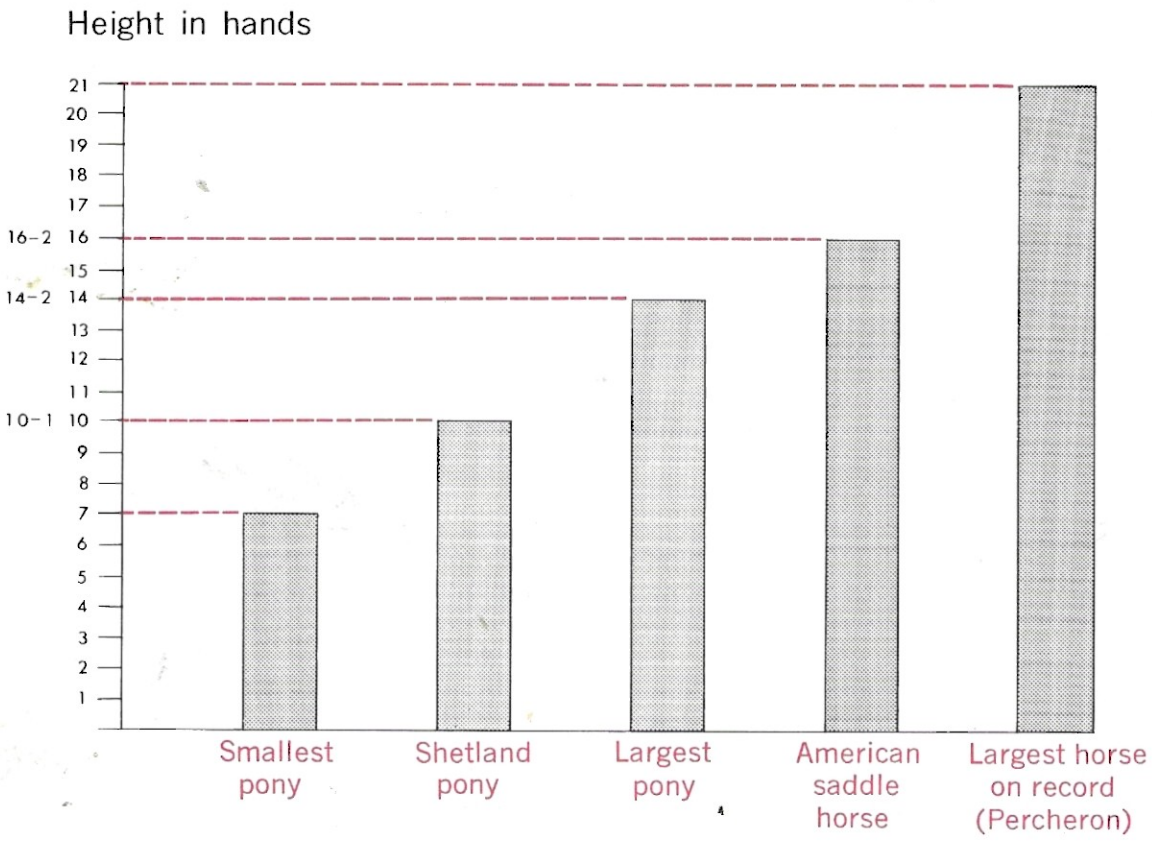
(A) $431\overline{)2586}$ → 6; -2586

(B) 500; $486\overline{)2670}$ → 5; -2430; 240

(C) 500; $450\overline{)1655}$ → 3; -1350; 305

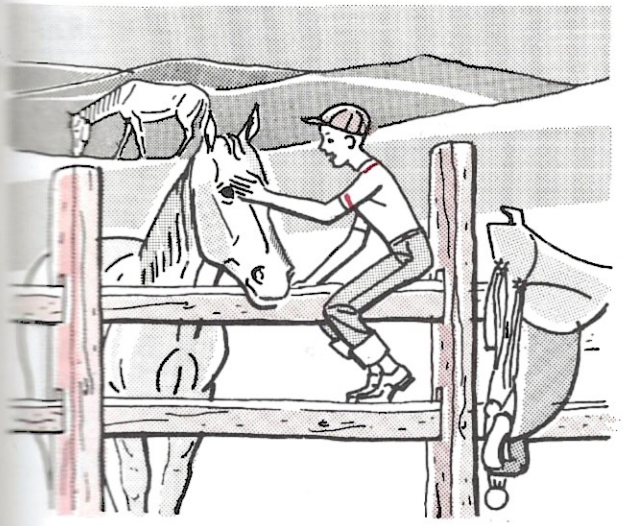
[A] $200\overline{)1800}$	[B] $600\overline{)3000}$	[C] $205\overline{)1562}$	[D] $595\overline{)2534}$
[E] $410\overline{)1640}$	[F] $823\overline{)7407}$	[G] $675\overline{)4350}$	[H] $621\overline{)1972}$
★[I] $423\overline{)10{,}152}$	★[J] $678\overline{)36{,}612}$	★[K] $237\overline{)17{,}785}$	★[L] $845\overline{)70{,}135}$

Horses



Horses are measured using a unit called a hand (4 inches). A horse's height is measured from the ground to the highest point of the ridge between its shoulder bones. If a horse is 16 hands and 2 inches tall, the height is written 16–2.

1. Give the height in inches of each horse mentioned above.
2. Give the height in feet (to the nearest foot) of each horse shown above.
3. Give the average height in inches of the horses mentioned above. Use the numbers you found in exercise 1.
4. The largest horse on record weighed 2995 pounds. This is 9 times as much as a small Shetland pony. Give the weight of the Shetland pony (to the nearest pound).



Type of horse	Weight (lb)
Pony	496
Saddle horse	1057
Light harness horse	1139
Heavy harness horse	1367
Draft horse	2169

This table gives 5 different types of horses and the weight of one horse of each type.

5. Find the average of the weights given in the table.

6. It has been said that a horse should be fed one pound of grain per day for each 100 pounds it weighs. If this rule is followed, about how many pounds of grain should be given each day to the draft horse whose weight is given in the table above?

7. The average life span of man, 69 years, is 3 times the average life span of a horse. What is the average life span of a horse?

8. A near-record high jump for a horse is 8 feet 2 inches. A near-record high jump for a man is 7 feet 4 inches. How much higher is the horse's jump?

9. A light horseshoe weighs 8 ounces. Give the total weight in pounds of the horseshoes needed to shoe a team of 6 horses.

★10. Fast horses can run about 38 miles per hour for a short time. If a horse runs at a rate of 30 miles per hour how far does it run in one minute?

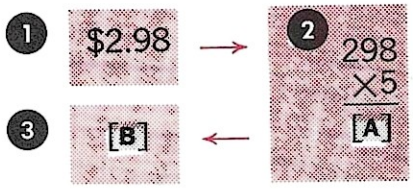
★11. A strong horse can lift 33,000 pounds 1 foot high in 1 minute. An engine that can do as much work as one strong horse is called a **1-horsepower** engine. How many pounds can a 34-horsepower engine lift 1 foot in 1 minute?

Money problems–multiplication and division

Study each example. Give what should go in each gray space.

A Ellen bought 5 books, each costing $2.98. What was the total cost? We use multiplication to solve this money problem.

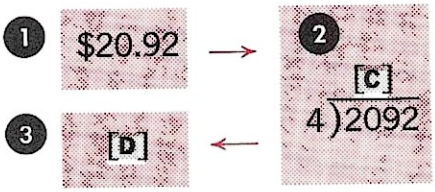
1. Think of the number of cents in $2.98.
2. Multiply by 5.
3. Write the answer using dollar notation.



Instead of writing the multiplication shown in part 2, you can write → $2.98 ×5 and shorten your work by **thinking about** the numbers 298 and 5 when multiplying. Then you can write the answer using dollar notation.

B Ann, Jan, Nan, and Fran received $20.92 from the sale of Christmas candy and cookies they had made. They divided the money so that each received the same amount. How much did each girl receive? We use division to solve this money problem.

1. Think of the number of cents in $20.92.
2. Divide by 4.
3. Write the answer using dollar notation.



Instead of writing the division shown in part 2, you can write $4\overline{)\$20.92}$ and think about the number 2092 as you divide. Then write the answer using dollar notation.

C A radio costs $18.00, including interest charges. If you pay $.75 a month, how many months does it take you to complete the payments? We use division to solve this money problem. Think about the numbers 75 and 1800 and find the quotient.

$$\$.75\overline{)\$18.00}\quad [E]$$

EXERCISES

1. Find the answers to these money problems.

[A] $\$6.75 \times 3$ [B] $\$.88 \times 6$ [C] $\$8.99 \times 8$ [D] $\$.35 \times 23$ [E] $\$1.39 \times 365$

[F] $5\overline{)\$2.95}$ [G] $7\overline{)\$44.38}$ [H] $30\overline{)\$87.00}$ [I] $65\overline{)\$22.10}$

[J] $\$.05\overline{)\$1.00}$ [K] $\$.50\overline{)\$7.50}$ [L] $\$.75\overline{)\$17.25}$ [M] $\$.25\overline{)\$20.00}$

2. Solve the problems.

[A] A jet plane carried 84 passengers from San Francisco to New York. The fare with tax was $160.90. How much money was received by the airline?

[B] Jane took 20 pictures with her new camera. 4 of the pictures were overexposed and were not developed. It cost $3.84 to develop the other pictures. What was the cost of each developed picture?

[C] Linda works at the school popcorn stand. She needs some coins in order to make change. How many quarters can she get for a $10 bill?

[D] A new bike cost $56.00. Jim received $12.50 for his old bike and paid the remainder in 6 payments. How much was each payment?

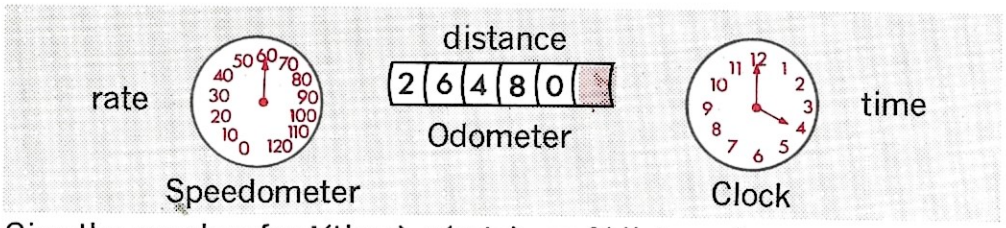
3. [A] A hamburger and a bag of potato chips cost $.50. Two hamburgers and a bag of potato chips cost $.85. What is the cost of a bag of potato chips?

[B] A dozen doughnuts and a loaf of bread cost $.64. Half a dozen doughnuts and a loaf of bread cost $.44. What is the price of a dozen doughnuts?

Think Think Think Think

A dozen cookies and a loaf of bread cost $.56. Half a dozen cookies and 2 loaves of bread cost $.58. What is the cost of a loaf of bread?

Short stories–time, rate, and distance



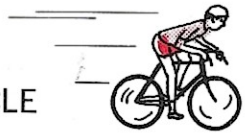
Give the number for **t**(time), **r**(rate), or **d**(distance).

1. WALK

[A] Slow: 3 miles. 3 hours. **r** miles per hour.

[B] Average: 3 miles per hour. 3 miles. Walked for **t** hours.

[C] Fast: 1 hour. 6 miles per hour. Went **d** miles.



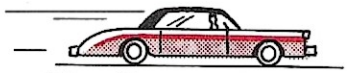
2. BICYCLE

[A] Slow: 5 hours. 15 miles. **r** miles per hour.

[B] Average: 10 miles per hour. 20 miles. Rode for **t** hours.

[C] Fast: 23 miles per hour. 2 hours. Went **d** miles.

3. AUTO



[A] Slow: 23 miles per hour. 115 miles. Took **t** hours.

[B] Average: 6 hours. 330 miles. **r** miles per hour.

[C] Fast: 139 miles per hour. 3 hours. Needed to go **d** more miles to finish the 500-mile race.

4. SHIP

[A] Slow: 105 miles. 7 hours. **r** miles per hour.

[B] Average: 32 miles per hour. 288 miles. Took **t** hours.

[C] Fast: 41 miles per hour. 76 hours. Traveled **d** miles.

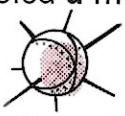
5. PLANE

[A] Slow: 570 miles. 6 hours. Traveled **r** miles per hour.

[B] Average: 365 miles per hour. 2555 miles in **t** hours.

[C] Fast: 1525 miles per hour. 4 hours. Traveled **d** miles.

6. SATELLITE



[A] Average (in earth's orbit): 17,546 miles per hour. 4 hours. Went **d** miles.

[B] Fast (escaping earth's orbit): 196,000 miles. 8 hours. **r** miles per hour.

★**7.** EARTH

Travels 595 million miles in about 365 days. Travels **r** miles per hour (rounded to nearest thousand).

Find the number

Find the number for **n** in each exercise.

1. **n** is 232 more than 649.
2. **n** is 358 less than 914.
3. **n** is 913 more than the sum of 432 and 956.
4. **n** is 387 less than the sum of 390 and 423.
5. **n** is 786 more than the difference of 3142 and 787.
6. The sum of 862 and **n** is 1465.
7. The product of 47 and 12 is 519 less than **n**.
8. The product of 86 and 35 is 1946 more than **n**.
9. The product of 37 and 29 is half as large as **n**.
10. The product of 46 and 54 is twice as large as **n**.
11. The product of 72 and 24 is 9 times as large as **n**.
12. The product of 15 and 15 is 25 times as large as **n**.
13. The product of 86 and **n** is 602.
14. The product of 72 and 4 is equal to the product of 32 and **n**.
15. The product of 40 and 7 is equal to the product of 56 and **n**.

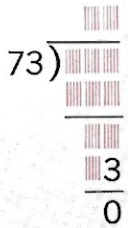
★16. The product of 28 and 54 is 6 times as much as the product of 36 and **n**.

Think Think Think Think

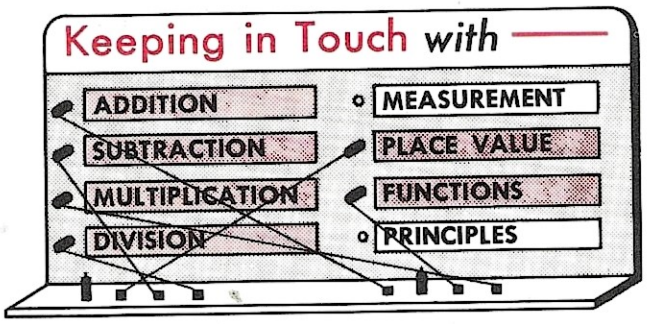
Find the number for **n** in each of the first four equations. Use a short cut to find the last four products.

$25 \times 25 = (20 \times 30) + n$
$35 \times 35 = (30 \times 40) + n$
$45 \times 45 = (40 \times 50) + n$
$55 \times 55 = (50 \times 60) + n$
$65 \times 65 =$?
$75 \times 75 =$?
$85 \times 85 =$?
$95 \times 95 =$?

Copy the problem. Give all the missing digits.



Keeping in Touch with



1. Solve the equations.

[A] $n \times 6 = 24$ [F] $8 \times t = 0$
[B] $8 \times r = 32$ [G] $n \times 11 = 66$
[C] $5 \times p = 35$ [H] $a \times 13 = 39$
[D] $q \times 9 = 54$ [I] $15 \times b = 75$
[E] $t \times 7 = 63$ [J] $8 \times q = 8$

2. Find the products.

[A] $10^1 \times 10^1$ [D] $10^1 \times 10^3$ [G] $5 \times 10 \times 4 \times 10$ [J] 50×80
[B] $10^1 \times 10^2$ [E] $10^2 \times 10^3$ [H] $6 \times 10^2 \times 3 \times 10$ [K] 90×600
[C] $10^2 \times 10^2$ [F] $10^3 \times 10^2$ [I] $8 \times 10^2 \times 9 \times 10^3$ [L] 800×400

3. Copy the sentence. Give the missing words and numbers.

[A] 438,614: The 8 in the _?_ place means $8 \times$ ▒▒.
[B] 697,102: The 6 in the _?_ place means $6 \times$ ▒▒.
[C] 73,602,549: The 7 in the _?_ place means $7 \times$ ▒▒.

4. Write the number that is:

[A] 10,000 more than one million. [B] 1 less than one million.

5. Write the ordinary base ten numeral for each exercise.

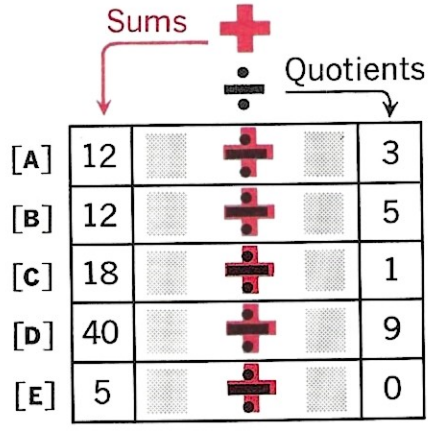
[A] 8×10^2 [C] 70×10^1 [E] $(8 \times 10^3) + (3 \times 10^2) + (4 \times 10) + 6$
[B] 6×10^3 [D] 80×10^2 [F] $(7 \times 10^5) + (4 \times 10^2) + 3$

6. Give the missing numbers for the table.

function rule: $n^2 + 5$

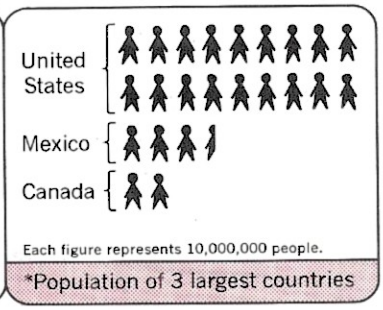
	n	$f(n)$
[A]	6	▒▒
[B]	10	▒▒
[C]	0	▒▒
[D]	▒▒	86
[E]	▒▒	54

★**7.** Give the pair of numbers for the gray spaces in each row.

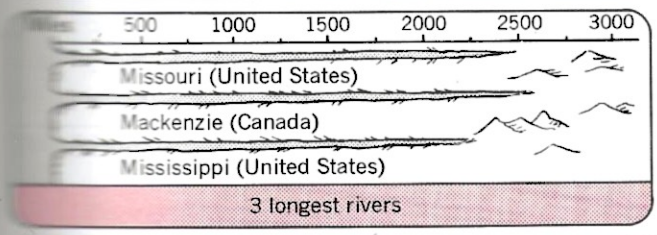


[A]	12	▒▒	3
[B]	12	▒▒	5
[C]	18	▒▒	1
[D]	40	▒▒	9
[E]	5	▒▒	0

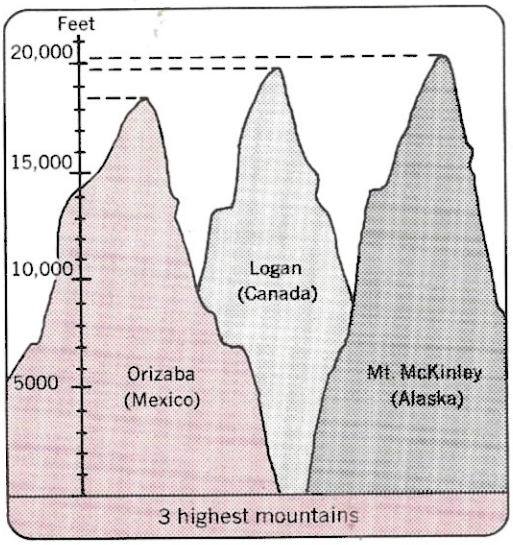
North America



*Population of 3 largest countries



3 longest rivers



3 highest mountains

Canada	3,851,113
U. S.	3,557,098
Mexico	760,373

3 largest countries (area in square miles)

New York	7,886,900
Chicago	3,811,400
Mexico City	3,301,760

3 largest cities (population*)

Superior (Canada and U. S.)	31,820
Huron (Canada and U. S.)	23,010
Michigan (U. S.)	22,400

3 largest lakes (area in square miles)

1. Use the graphs above to give:
   - [A] The populations of the 3 largest countries in North America (to the nearest 5 million people).
   - [B] The lengths of the 3 longest rivers (to the nearest hundred miles).
   - [C] The heights of the 3 highest mountains (to the nearest 1000 feet).
   - [D] An estimate of the difference in height between Mt. McKinley and Mt. Logan.
2. Use the tables above to give:
   - [A] The areas of the 3 largest countries (rounded to the nearest 500,000).
   - [B] The populations of the 3 largest cities (rounded to the nearest million).
   - [C] The average area of the 3 largest lakes (to the nearest whole number).
3. The average number of persons per square mile in North America is 28. The area of North America is 9,435,000 square miles. About what is the population of North America?

*Population figures from a recent year.

Chapter review

1. Find the sums, products, differences, or quotients.

[A] $\begin{array}{r} 5764 \\ +3847 \\ \hline \end{array}$ [B] $\begin{array}{r} 861 \\ -463 \\ \hline \end{array}$ [C] $\begin{array}{r} 3947 \\ -1698 \\ \hline \end{array}$ [D] $\begin{array}{r} 653 \\ \times 25 \\ \hline \end{array}$

[E] $876 + 39 + 5483 + 9$ [F] $683 + 4765 + 98 + 17$

[G] $7\overline{)6478}$ [H] $7004 - 39$ [I] 37×56 [J] $50\overline{)3076}$

[K] $\begin{array}{r} 566 \\ \times 207 \\ \hline \end{array}$ [L] $\begin{array}{r} 8070 \\ -4293 \\ \hline \end{array}$ [M] $62\overline{)434}$ [N] $79\overline{)635}$

[O] $\begin{array}{r} 308 \\ \times 72 \\ \hline \end{array}$ [P] $35\overline{)2205}$ [Q] $\begin{array}{r} 6003 \\ -5786 \\ \hline \end{array}$ [R] $74\overline{)18{,}944}$

[S] $57\overline{)874}$ [T] $16\overline{)12{,}675}$ [U] $92\overline{)556{,}947}$ [V] $486\overline{)3786}$

2. Give the missing numbers.

[A] $4^2 =$ ▨ [B] $5^3 =$ ▨ [C] $6^4 =$ ▨ [D] $3^5 =$ ▨

3. Mark **true** or **false**.

[A] $3^2 = 2^3$ [B] $2^5 > 5^2$

4. Find the arithmetic mean of each set of numbers.

[A] {562, 385, 419, 646}

[B] {88, 107, 365, 419, 136}

5. Find the total amounts.

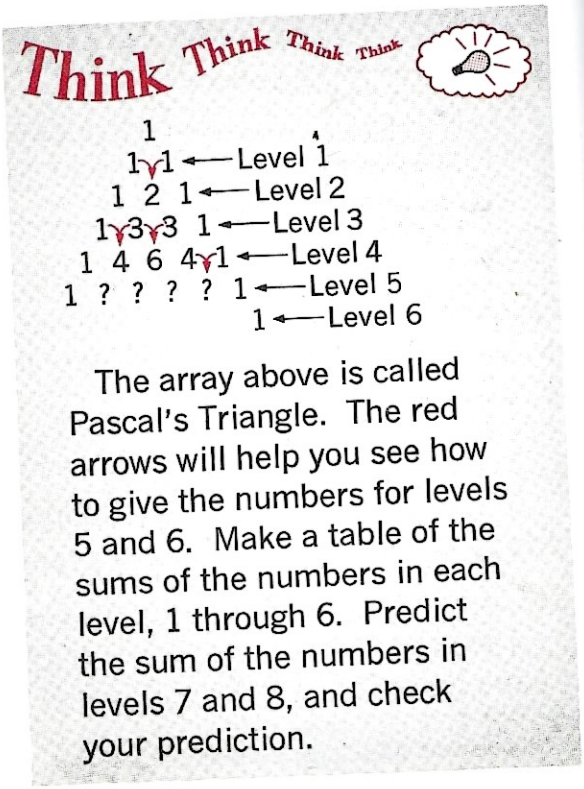
[A] $\begin{array}{r} \$18.75 \\ 26.99 \\ \hline \end{array}$ [B] $\begin{array}{r} \$23.64 \\ 8.98 \\ \hline \end{array}$

6. Find the differences in the amounts.

[A] $\begin{array}{r} \$65.50 \\ 65.37 \\ \hline \end{array}$ [B] $\begin{array}{r} \$20.00 \\ 9.34 \\ \hline \end{array}$

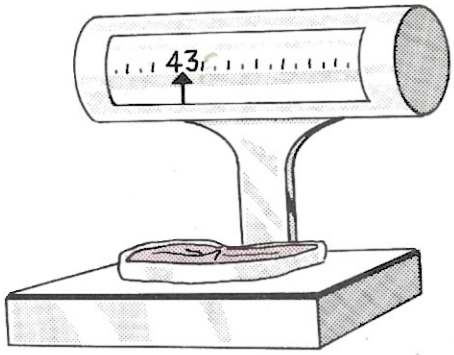
7. Solve these money problems.

[A] $\begin{array}{r} \$3.49 \\ \times 4 \\ \hline \end{array}$ [B] $\begin{array}{r} \$.87 \\ \times 25 \\ \hline \end{array}$ [C] $9\overline{)\$21.06}$



The array above is called Pascal's Triangle. The red arrows will help you see how to give the numbers for levels 5 and 6. Make a table of the sums of the numbers in each level, 1 through 6. Predict the sum of the numbers in levels 7 and 8, and check your prediction.

8. A piece of meat weighs 43 ounces.



[A] How many ounces less than 3 pounds is this?

[B] What is the total cost of 3 pounds of meat at $1.19 per pound?

[C] 5 pounds of another type of meat costs $6.15. How much more or less does this meat cost per pound than the meat in part B?

9. A plane flew at an average speed of 342 miles per hour for 7 hours. How far did it fly?

10. Mr. Smith bought a television set. The cost plus interest charges was $340.08. If he pays for it in 12 equal payments, how much will each payment be?

11. Here is part of Jim's record of the time it takes him to deliver papers each day. He spent a total of 370 minutes delivering papers during this week. How long did it take him to deliver papers on Saturday?

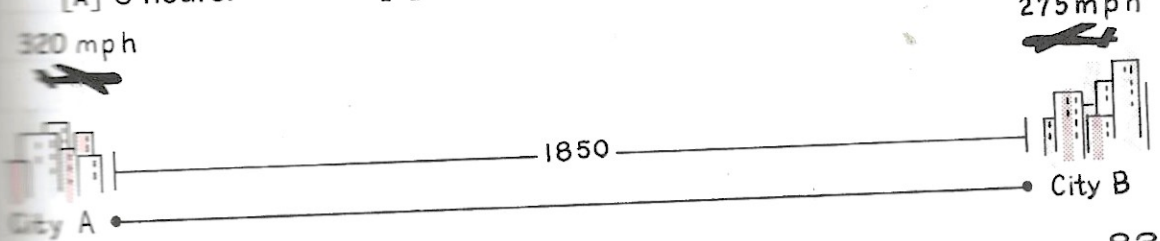
Days	M	T	W	Th	F	S
Minutes	55	65	65	60	50	?

*12. The chart shows the number of minutes Sally practiced the piano during one week. How many minutes must she practice on Saturday in order to have practiced an average of 35 minutes a day?

Days	M	T	W	Th	F	S
Minutes	30	25	35	40	30	?

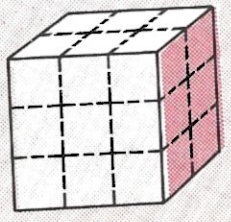
*13. If the planes fly toward each other at the speed given, how far apart will they be after:

[A] 3 hours? [B] 4 hours?



Volume

▶ We count cube units to find volume. Using the unit shown, we say that the volume of the figure is 18.

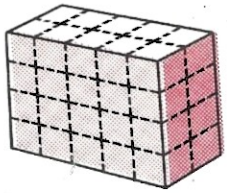


Unit

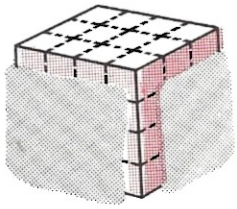
EXERCISES

1. Give the volume for each figure. Use the units given.

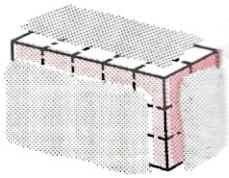
[A]



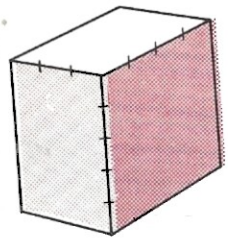
[B]



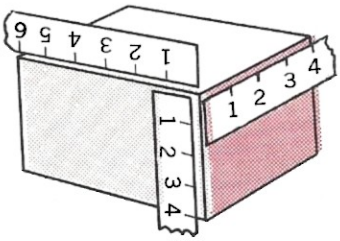
[C]



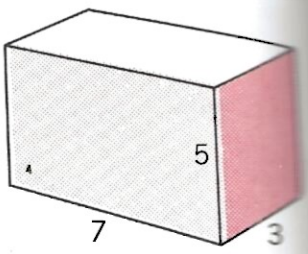
[D]



[E]



[F]



2. Make a table like the one on the right and give the missing numbers. The table refers to exercise 1.

Figure	l(↔)	w(↗)	h(↕)	V(volume)
[A]	5	2	4	
[B]	4	3		
[C]	5		3	
[D]		4		
[E]				
[F]				

3. Give the volumes of the rectangular **prisms** (boxes). **l** = length; **w** = width; **h** = height

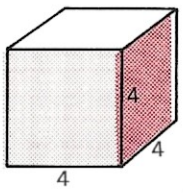
[A] **l** = 9, **w** = 3, **h** = 6

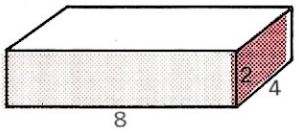
[B] **l** = 12, **w** = 2, **h** = 8

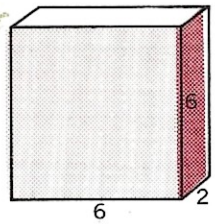
[C] **l** = 19, **w** = 13, **h** = 14

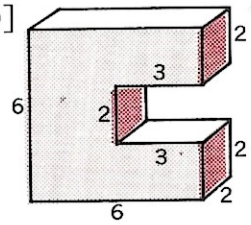
4. Explain this formula: **V** = **l** × **w** × **h**

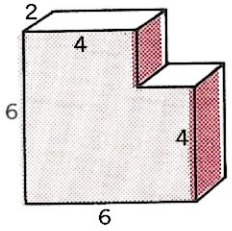
5. Give the volume and surface area of each figure.

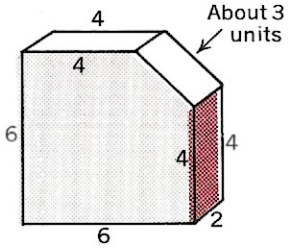
[A] 

[B] 

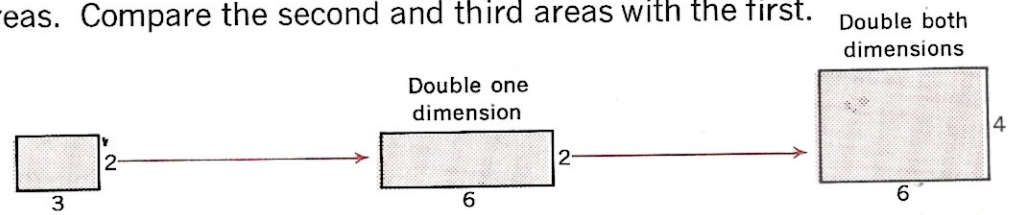
[C] 

★ [D] 

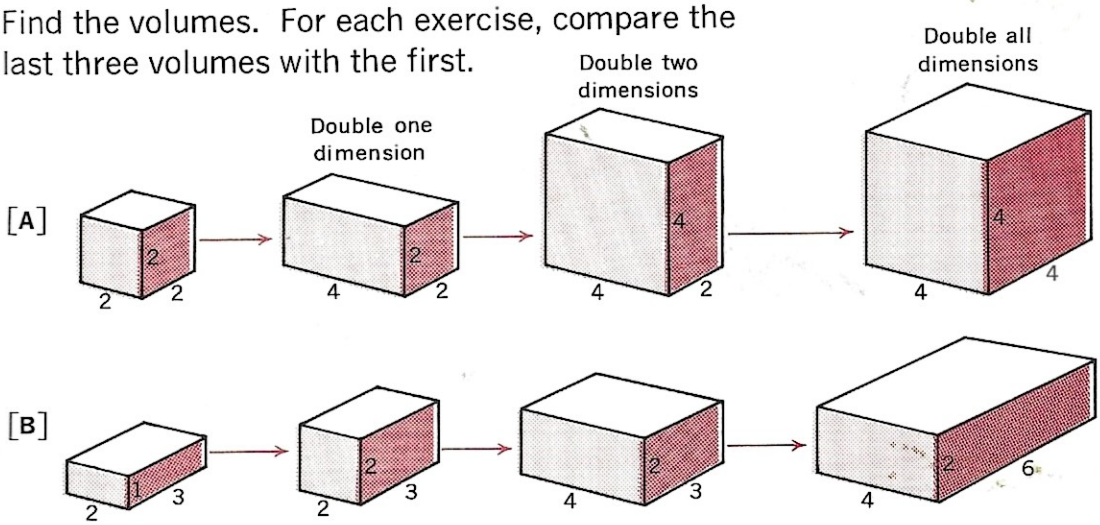
★ [E] 

★ [F] 

6. Length, width, and height are sometimes called **dimensions**. Find the areas. Compare the second and third areas with the first.



7. Find the volumes. For each exercise, compare the last three volumes with the first.

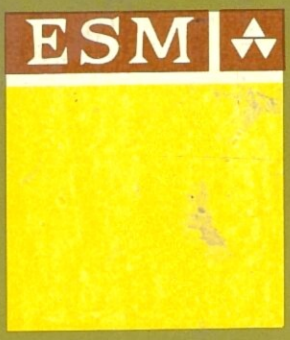


8. If each dimension of a rectangle is doubled, the area is _?_ times as large.

9. If each dimension of a box is doubled, the volume is _?_ times as large.

★ 10. What happens to the volume of a box if each dimension is tripled?

EICHOLZ • O'DAFFER • BRUMFIEL • SHANKS



ADDISON-WESLEY PUBLISHING COMPANY, INC.
Reading, Massachusetts / Palo Alto / London